K23N 0421

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, October 2023 (2022 Admission Onwards) STATISTICS WITH DATA ANALYTICS MST1 C02 : Probability Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. Define a probability space. What is the monotone property of a probability measure ?

2. Define the concept of limit points of a set. Prove that every accumulation point is a limit point, but not vice versa.

3. Define the cumulative distribution function (CDF) of a random variable. Show that the limit of the CDF as (x) approaches negative infinity is 0.

4. State the continuity theorem of characteristic functions.

5. Define almost sure convergence of a sequence of random variables.

6. State the Helly-Bray theorem, describing the convergence of the sequence of distribution functions.

7. Define the strong law of large numbers.

8. Compare and contrast the Classical Central Limit Theorem and the De Moivre-Laplace Central Limit Theorem. **(8×2=16)**

P.T.O.

PART – B

Answer **any 4** questions. **Each** question carries **4** marks.

9. Define the minimal sigma-field generated by a collection of sets. Show that the power set of a set is a sigma field.

10. Define a random variable. Prove that the sum of two random variables is also a random variable.

11. How probability mass function (pmf) is derived from a CDF ? Prove that a CDF is non-decreasing and right-continuous.

12. State and prove Chebyshev's inequality for any random variable.

13. Prove that convergence in probability implies convergence in distribution.

14. Let $\{Y_n\}$ be a sequence of independent Poisson random variates with mean λ_n. Then for $n \to \infty$, if $\sum \frac{\lambda_k}{n} \to 0$ then, show that $\{Y_n\}$ obey WLLN. **(4×4=16)**

PART – C

Answer **any 4** questions. **Each** question carries **12** marks.

15. A) Define the limit superior and limit inferior of a sequence of sets A_n. Prove that $\limsup A_n = \bigcap_{\{k=1\}}^{\infty} \bigcup_{\{n=k\}}^{\infty} A_n$ and $\liminf A_n = \bigcup_{\{k=1\}}^{\infty} \bigcap_{\{n=k\}}^{\infty} A_n$.

    B) Prove that for any sequence of events (A_n), $\limsup A_n$ and $\liminf A_n$ are both events and $\limsup A_n \subseteq \liminf A_n$.

16. Let X be an RV with EX = 0 and $var(X) = \sigma^2$. Then show that

    A) $P\{X \geq x\} \leq \frac{\sigma^2}{\sigma^2 + x^2}$ if $x > 0$

    B) $P\{X \geq x\} \geq \frac{x^2}{\sigma^2 + x^2}$ if $x < 0$.

17. State and prove the basic inequality.

18. A) Show that the characteristic function is always bounded by 1.

    B) Prove that the characteristic function of a random variable is uniformly continuous.

19. A) Prove that if (X_n) converges to (X) almost surely and (Y_n) converges to (Y) almost surely, then $(X_n + Y_n)$ converges to $(X + Y)$ almost surely.

    B) Show that if (X_n) converges to (X) in probability and $(g(x))$ is a continuous function, then $(g(X_n))$ converges to $(g(X))$ in probability.

20. A) State and prove the weak law of large numbers due to Bernoulli for a sequence of independent identically distributed random variables.

    B) Let $X_1, X_2, ..., X_n$ be independent and identically distributed random variables with PMF $P(X_i = k) = 1/2^k$ for $k = 1, 2, 3, ...$ check if the weak law of large numbers due to Khintchine is satisfied for this sequence. **(4×12=48)**