K23U 0516

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Find the null space and range space of the zero transformation from R^3 to R^3.
2. Write a subspace of $M_{n \times n}(F)$.
3. What is the dimension of C over R ?
4. State Sylvester's law of nullity.
5. Give an example for an infinite dimensional vector space.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Let $T : R^2 \to R^2$ defined by $T(x, y) = (1, y)$. Is T linear ?
7. Prove that in any vector space V, $0x = 0$, for each $x \in V$.
8. State Dimensional theorem.
9. Let $T : R^2 \to R^3$ defined by $T(x, y) = (x + 7y, 2y)$. Write the matrix of T with respect to the standard ordered bases of R^2 and R^3.
10. If – 2 and 2 are eigen values of a square matrix A, then what are the eigen values of A′, transpose of A ?

P.T.O.

11. Let $T : F^2 \to F^2$ be a linear transformation defined by $T(x, y) = (1 + x, y)$. Find $N(T)$.

12. Determine whether $\{(2, -4, 1), (0, 3, -1), (6, 0, -1)\}$ form a basis for R^3.

13. Define an elementary matrix.

14. Let A be a 2×2 orthogonal matrix with 3 as an Eigen value. What will be the other Eigen value of A ?

15. Give an example for a linear transformation $T : F^2 \to F^2$ such that $N(T) = R(T)$.

16. State Cayley Hamilton theorem.

PART – C

Answer **any 4** questions. **Each** question carries **four** marks.

17. Define a vector space.

18. Prove that $P_n(F)$ is a vector space.

19. Prove that any intersection of subspaces of a vector space V is a subspace of V.

20. Prove that $\text{rank}(AA') = \text{rank}(A)$.

21. Find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 3 & 4 & 5 & 2 \\ 2 & 3 & 4 & 0 \end{bmatrix}$.

22. Let W be a subspace of a finite dimensional vector space V. Then prove that W is finite dimensional and $\dim W \leq \dim V$. Moreover if $\dim W = \dim V$ then prove that $V = W$.

23. Let $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. Find A^{-1} using Cayley Hamilton theorem.

PART – D

Answer **any 2** questions. **Each** question carries **six** marks.

24. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$ into normal form and hence find the rank.

25. Solve the system of equations
$x + 3y - 2z = 0,\ 2x - y + 4z = 0,\ x - 11y + 14z = 0.$

26. Find the Eigen values and Eigen vectors of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

27. State and prove replacement theorem.