K23U 1126

Reg. No. : .....

Name : .....

IV Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2023
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** out of five questions. **Each** question carries 1 mark.

1. Define a prime number.
2. State Euclidian lemma.
3. When we can say that the existence of solution Diophantine equation of the form ax + by = c ?
4. State Wilson's theorem.
5. Show that for any integers a, n, a ≡ a(modn). **(4×1=4)**

PART – B
(Short Essay Type)

Answer **any eight** out of eleven questions. **Each** question carries **2** marks.

6. If a|b, then show that a|bc, for any integer c.
7. Find gcd(24, 138) using Euclidian algorithm.
8. Show that, if p is a prime and p|ab, then p|a or p|b.
9. If a ≡ b(mod n), prove that gcd(a, n) = gcd(b, n).

P.T.O.

10. State Fermat's little theorem.

11. Evaluate $\int_{-1}^{1} 3x^2\sqrt{x^3+1}\,dx$.

12. Find the area of the region bounded above by $y = x + 6$ bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$.

13. Define volume problem.

14. Find the volume of the solid that is obtained when the region under the curve $y = \sqrt{x}$, over the interval [1, 4] is revolved about x-axis.

15. Find the arc length of the spiral $r = e^{\theta}$ between $\theta = 0$ and $\theta = 2 + \pi$.

16. Find the area of the surface generated by $y = 7x$, $0 \le x \le 1$, revolving about x-axis. **(8×2=16)**

PART – C
(Essay Type)

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Solve the Diophantine equation $172x + 20y = 1000$.

18. Express 6 as a linear combination of 12378 and 3054.

19. Find $2^{340} \pmod{341}$.

20. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

21. Find the area of the region enclosed by the rose curve $r = \cos 2\theta$.

22. Find the arc length of the curve $y = x^{\frac{3}{2}}$, from (1,1) to $(2, 2\sqrt{2})$.

23. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis. **(4×4=16)**

PART – D
(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. Let x_0, y_0 is any particular solution of the Diophantine equation of the form $ax + by = c$, then show that all other solutions can be represented by $x = x_0 + \left(\frac{b}{d}\right) t$, $y = y_0 + \left(\frac{a}{d}\right) t$, where $d = \gcd(a, b)$.

25. Use Euler's theorem, evaluate $2^{100000} \pmod{77}$.

26. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, using integration.

27. Find the volume of the solid generated when the region under $y = x^2$ over the interval $[0, 2]$ is revolved about the line $y = -1$. **(2×6=12)**