K23P 1251

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Regular) Examination, October 2022
(2022 Admission)
STATISTICS WITH DATA ANALYTICS
MST1C01 : Mathematical Methods for Statistics

Time : 3 Hours Max. Marks : 80

PART – A

(Answer **all** questions. **Each** question carries **2** marks).

1. Define basis and dimension of a vector space.

2. Show that determinant of a matrix is equal to the product of its eigenvalues.

3. Show that rank of an idempotent matrix is equal to its trace.

4. Define positive definiteness of a matrix. Give one example.

5. Show that the function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, has a removable discontinuity at the origin.

6. Define open and closed sets in the context of metric space.

7. State Cauchy's criterion for uniform convergence of a sequence of functions.

8. Show that the sequence $f_n(x) = \frac{1}{x+n}$ is uniformly convergent on any interval $[0, b]$, $b > 0$. **(8×2=16)**

PART – B

(Answer **any four** questions. **Each** question carries **4** marks).

9. Solve the following equations using Gaussian elimination method.

   $x + 2y + 3z = 1$

   $2x + 5y + 8z = 4$

   $3x + 8y - 13z = 7$

P.T.O.

10. Show that the system of equation AX = B is consistent if and only if r(A : B) = r(A).

11. Compute the eigenvalues and eigenvectors of the matrix.

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

12. Show that if f is monotone on [a, b] and α is continuous on [a, b], then $f \in R(\alpha)$.

13. State and prove Weirstrass M-test.

14. Show that $\lim_{(x, y) \to (0, 0)} \frac{2xy^2}{x^2 + y^4}$ does not exist. **(4×4=16)**

PART – C

(Answer **any four** questions. **Each** question carries 12 marks).

15. i) Prove that rank of the product of two matrices cannot exceed the rank of any one of them.

    ii) Let V be a vector space with dimension n. Show that any linearly independent set in V can be extended to a basis of V.

16. i) State and prove Cayley-Hamilton Theorem.

    ii) Given $A = \begin{bmatrix} 1 & 4 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$. Show that the matrix A satisfies the characteristic equation and hence find A^{-1}.

17. State and prove the necessary and sufficient condition that a real quadratic form X′ AX is positive definite.

18. Define Reimann-Stieltjes integral and state and prove any two properties of it.

19. Define uniform convergence of a sequence of functions. Assume that $f_n \to f$ uniformly on S. If each f_n is continuous at a point c of S, then show that the limit function f is also continuous at c.

20. State and prove Taylor's theorem for the function of several variables. **(4×12=48)**