K21U 1537

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2017-'18 Admns.)
CORE COURSE IN MATHEMATICS
5B09 MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

PART – A

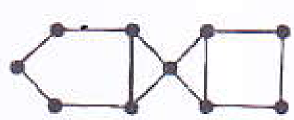
Answer **all 4** questions : **(1×4=4)**

1. Draw a graph on 4 vertices having a cut edge. Mark the cut edges.
2. Sketch 2 non isomorphic trees on 4 vertices.
3. Plot a tournament on 4 vertices.
4. Sketch a symmetric digraph on 4 vertices.

PART – B

Answer **any 8** questions : **(2×8=16)**

5. If G is simple and $\delta \geq \frac{n-1}{2}$, then prove that G is connected.
6. By considering two graphs G_1 and G_2 on 3 vertices, draw $G_1 \cup G_2$ and $G_1 \cap G_2$.
7. If an edge e = xy of a connected graph G belongs to no cycle of G, prove that e is a cut edge of G.
8. Find the cut edges and the cut vertices of the graph given below.



9. Define the terms vertex connectivity and edge connectivity of a graph.

P.T.O.

10. By drawing a connected graph on 5 vertices, identify a spanning tree in it.
11. Justify the claim : A subset S of V is independent if and only if V – S is a covering of G.
12. Is the Wheel graph W_5 Hamiltonian ? Justify your claim.
13. Define a strict digraph. Give an example of a strict digraph on 4 vertices.
14. Give an example of a strong digraph. Justify your answer.

PART – C

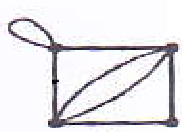
Answer **any 4** questions : **(4×4=16)**

15. Show that if G is a self complementary graph of order n, then $n \equiv 0$, 1 (mod 4).
16. In any graph G, prove that number of vertices of odd degree is even.
17. Prove that any connected graph contains a spanning tree.
18. Show that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.
19. For a graph G, prove that $\alpha' + \beta' = n$.
20. Graphically explain the Königsberg bridge problem. Explain why the solution is not possible.

PART – D

Answer **any 2** questions : **(6×2=12)**

21. Show that a graph G is bipartite if and only if it contains no odd cycle.
22. Find $\tau(G)$ of the graph.



G

23. Prove that a graph G is Eulerian if and only if each edge e of G belongs to odd number of cycles of G.
24. Prove that every tournament contains a directed Hamiltonian path.
