K25FY2463 B

Second Semester FYUGP Mathematics Examination
APRIL 2025 (2024 Admission onwards)
KU2DSCMAT113 (SET THEORY, NUMBER THEORY, INTEGRAL CALCULUS AND FOURIER SERIES)
(DATE OF EXAM: 30-4-2025)

Time : 120 min   Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Define **empty set**. Give an example for empty set.   3

2. (a) Define symmetric difference of two sets A and B.
   (b) Draw the Venn diagram to represent the symmetric difference of A and B.   3

3. Define union and intersection of two sets.   3

4. Compute $\int_0^{\frac{\pi}{2}} \sin^3 x \, dx$, using reduction formula.   3

5. Evaluate $\int \cos^4 x \, dx$, using reduction formula.   3

6. What is the fundamental period of the function $f(x) = \cos 3x$?   3

7. If $f(x)$ is an even function, write its Fourier series of period 2π?   3

8. If $f(x)$ is an even function, write its Fourier series of period $2L$?   3

Part B (Answer any 4 questions. Each carries 6 marks)

9. For the set $A = \{x \mid x$ is an even number less than $10\}$, write the set explicitly and list all the subsets of A.   6

10. For the set $A = \{x \mid x$ is a positive integer less than or equal to $10\}$ and the set $B = \{x \mid x$ is a multiple of 3 less than or equal to $10\}$, answer the following:
    (a) Write the sets A and B.
    (b) Find the power set of B.
    (c) Calculate $|A|, |B|$ and $|P(B)|$.   6

11. Let $U = \{1,2,3,4,5,6,7,8,9\}, A = \{1,2,4,6,8\}, B = \{2,4,5,9\}$, $C = \{x \mid x$ is a positive integer and $x^2 \le 16\}$, and D = $\{7,8\}$. Compute
    (a) $A \cup B \cup C$
    (b) $A \cap (\overline{C} \cup D)$
    (c) $B \oplus C$.   6

12. Determine the Fourier series coefficient a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of the function $f(x) = \begin{cases} 1 & -\pi < x < 0 \\ -1 & 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.   6

13. Determine the Fourier series coefficient b_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ of the function $f(x) = 1 + x$ on $[-\pi, \pi]$.   6

14. Obtain the Fourier coefficients a_0 and a_n in the Fourier series expansion $a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x \right)$ of the 2-periodic function defined by:

$$f(x) = \begin{cases} 0, & \text{when } -1 < x < 0 \\ 1, & \text{when } 0 < x < 1. \end{cases}$$

   6

Part C (Answer any 2 question(s). Each carries 14 marks)

15. Find $d = GCD(a, b)$ and write d as $sa + tb$ where $a = 108$, $b = 60$.   14

16. Find $d = GCD(a, b)$ and write d as $sa + tb$ where $a = 60$, $b = 100$.   14

17. Evaluate $\int \cos^{12} x \, dx$.   14