K21U 4554

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B09MAT : Vector Calculus

Time : 3 Hours Max. Marks : 48

PART – A

Short Answer

(Answer **any four**) **(1×4=4)**

1. Find the parametric equations for the line through (3, 1, 2) and (2, 1, 6).
2. State and prove chain rule for vector functions.
3. Find the directional derivative of $F(x, y, z) = xy^2 - 4x^2y + z^2$ at (1, –1, 2) in the direction of $6i + 2j + 3k$.
4. Evaluate $\oint_C x\,dx$ where C is the circle $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$.
5. If $F = (x^2y^3 - z^4)\,i + 4x^5y^2z\,j - y^4z^6\,k$ find curl F.

PART – B

Short Essay

(Answer **any eight**) **(2×8=16)**

6. A helicopter is to fly directly from a helipad at the origin in the direction of the point (1,1,1) at a speed of 60 ft sec. What is the position of the helicopter after 10 sec ?
7. A projectile is launched from ground level with an initial speed $v_0 = 768$ ft/s. at an angle $\theta = 30°$. What is the range and maximum height attained by the projectile ?

P.T.O.

8. Show that if $u = u_1 i + u_2 j + u_3 k$ is a unit vector, then the arc length parameter along the line $r(t) = (x_0 + tu_1)\, i + (y_0 + tu_2) j + (z_0 + tu_3)\, k$ from the point $P_0(x_0, y_0, z_0)$ where $t = 0$ is t itself.

9. Find the linearization of $f(x, y) = x^2 - xy + x^5y^2 + y^4$ at the point (2, 1).

10. Find the local extreme values of $f(x, y) = y^2 - x^2$.

11. Find the least squares line for the points (0, 1), (1, 3), (2, 2), (3, 4), (4, 5).

12. Prove the orthogonal gradient theorem.

13. A coil spring lies along the helix $r(t) = 2\cos t\, i + 2\sin t\, j + tk$, $0 \le t \le 2\pi$ with constant density δ. Find the spring's mass and center of mass, and its moment of inertia and radius of gyration about the z-axis.

14. Find work done by force $F = yz\, i + xz\, j + xy\, k$ acting along the curve given by $r(t) = t^3\, i + t^2 j + tk$ from $t = 1$ to $t = 3$.

15. Prove closed property of conservative fields.

16. Show that $ydx + xdy + 4dz$ is exact and evaluate the integral $\int ydx + xdy + 4dz$ over the line segment from (1, 1, 1) to (2, 3, – 1).

PART – C

Essay

(Answer **any four**) **(4×4=16)**

17. The position of a moving particle is given by $r(t) = 2\cos t\, i + 2\sin t\, j + 3tk$. Find the tangential, normal and binormal vectors. Also determine the curvature.

18. A delivery company accepts only rectangular boxes the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 in. Find the dimensions of an acceptable box of largest volume.

19. Find the maximum and minimum values of the function $f(x, y) = 3x - y$ on the circle $x^2 + y^2 = 4$ using Lagrange multiplier's.

20. Given $F(x, y) = (y^2 - 6xy + 6)\, i + (2xy - 3x^2 - 2y)\, j$. Determine a potential function for F.

21. How are the constants a, b and c related if the following differential form is exact ? $(ay^2 + 2czx)\, dx + y(bx + cz)\, dy + (ay^2 + cz^2)\, dz$.

22. Calculate the outward flux of the field $F(x, y) = xi + 3y^2 j$ across the square bounded by the lines $x = \pm 2$ and $y = \pm 2$.

23. Find the surface area of sphere of radius a.

PART – D

Long Essay

(Answer **any two**) **(2×6=12)**

24. a) Show that the curvature of a circle with radius a is $^1/_a$.

    b) Find and graph the osculating circle of the parabola $y = x^2$ at the origin.

25. a) State Taylor's formula for f(x, y) at the origin and at point (a, b).

    b) Find a quadratic approximation to $f(x, y) = \sin x \sin y$ near the origin. How accurate is the approximation if $|x| \le 0.2$ and $|y| \le 0.4$?

26. a) Find the flux and circulation of the field $F(x, y) = (x - y)\, i + xj$ around the circle $x^2 + y^2 = 1$.

    b) Evaluate $\oint_C (x^2 - y^2)\, dx + (2y - x)\, dy$ where C consists of the boundary of the region in the first quadrant that is bounded by the graphs of $y = x^2$ and $y = x^3$.

27. a) Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2$, $z \ge 0$ by the cylinder $x^2 + y^2 = 1$.

    b) Evaluate $\oint_C zdx + xdy + ydz$ where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counter clockwise as viewed from above.