K21U 4553

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021

(2019 Admn. Only)

CORE COURSE IN MATHEMATICS

5B08 MAT : Differential Equations and Laplace Transforms

Time : 3 Hours

Max. Marks : 48

PART – A

(Short Answer)

Answer **any four** questions. **Each** question carries **1** mark.

1. Verify that $y = e^{2x^2}$ is a solution of the ODE $y' - 4xy = 0$.
2. Give an example of a first order nonlinear ODE.
3. Find a basis of solutions of the ODE $y'' - 4y = 0$.
4. What is Euler-Cauchy equations ?
5. State convolution theorem. **(4×1=4)**

PART – B

(Short Essay)

Answer **any eight** questions. **Each** question carries **2** marks.

6. Solve the initial value problem $y' = 6y$, $y(0) = 2$.
7. Does the initial value problem $xy' = y - 1$ has a unique solution ? Justify.
8. Solve the IVP $y' = -4x/y$, $y(2) = 3$.
9. Find the general solution of $y' + ky = e^{-kx}$.
10. Find the general solution of $4y'' - 25y = 0$.

P.T.O.

11. Factor $P(D) = D^2 - 3D - 40I$ and solve $P(D)y = 0$.

12. Find the general solution of $x^2y'' - 5xy' + 9y = 0$.

13. Find the Wronskian of cos 6x and sin 6x.

14. Find the inverse transform f(t) of $F(s) = \frac{e^{-s}}{s^2+4} + \frac{e^{-2s}}{s^2+1} + \frac{e^{-3s}}{(s+2)^2}$.

15. Find the Laplace transform of tsin2t.

16. Find the inverse transform of $\frac{1}{s(s^2+9)}$. **(8×2=16)**

PART – C
(Essay)

Answer **any four** questions. **Each** question carries **4** marks.

17. Solve the IVP $e^{2x}(2\cos y\,dx - \sin y\,dy) = 0$, $y(0) = 0$.

18. Find the general solution of $y' = 1/(6e^y - 2x)$.

19. Solve $y'' + y' = 0$ by reducing it to first order.

20. Solve the IVP $y'' + y' - 6y = 0$, $y(0) = 10$, $y'(0) = 0$.

21. Solve the nonhomogeneous ODE $y'' + y = \sec x$.

22. Find the Laplace transform of the function $f(t) = \begin{cases} 2, & \text{if } 0 < t < 1 \\ \frac{1}{2}t^2, & \text{if } 1 < t < \frac{1}{2}\pi \\ \cos t, & \text{if } t > \frac{1}{2}\pi. \end{cases}$

23. Solve the IVP $y'' + 3y' + 2y = \delta(t-1)$, $y(0) = 0$, $y'(0) = 0$ by Laplace transform. **(4×4=16)**

PART – D
(Long Essay)

Answer **any two** questions. **Each** question carries **6** marks.

24. Solve $2xyy' = y^2 - x^2$ by reducing it to variable separable form.

25. Solve the IVP $(e^{x+y} + ye^y)\,dx + (xe^y - 1)\,dy = 0$, $y(0) = -1$.

26. Solve the initial value problem $y'' - 6y' + 9y = e^{3x}$, $y(0) = 1$, $y'(0) = 1$.

27. Solve the integral equation $y(t) - \int_0^t (1+\tau)\,y(t\tau)\,d\tau = 1 - \sinh t$ by Laplace Transform. **(2×6=12)**
