K25U 1434

Reg. No. : .....

Name : .....

Second Semester B.Sc. A.I. and M.L. Degree (C.B.C.S.S. – OBE – Supplementary/Improvement) Examination, April 2025
(2023 Admission)
Complementary Elective Course
2C02MAT AIML : INTEGRATION AND LINEAR ALGEBRA

Time : 3 Hours Max. Marks : 40

PART – A
Short Answer

Answer **all** questions from this Part. **Each** question carries 1 mark. **(6×1=6)**

1. Find $\lim_{x \to 0, y \to 0} \frac{xy}{x^2 + y^2 + 1}$.
2. Let $z = x^5 + y^3 - 6bxy^2$. Find $\frac{\partial z}{\partial x}$.
3. Write the reduction formula for $\int \cos^n x dx$.
4. Give an example of a vector space.
5. Define characteristic equation of the matrix A.
6. When can you say that a matrix is diagonalizable ?

PART – B
Short Essay

Answer **any six** questions from this Part. **Each** question carries **2** marks. **(6×2=12)**

7. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$. Find $\frac{\partial^2 u}{\partial y \partial x}$.
8. Given $u = \sin\left(\frac{x}{y}\right)$, $x = e^t$ and $y = t^2$, find $\frac{du}{dt}$ as a function of t, using chain rule.
9. Find $\int \sin^4 x dx$.
10. Find $\int \tan^3 x dx$.

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11. Define dimension of a vector space. Give an example of a two dimensional vector space.

12. Define linear transformation. Give an example of a linear transformation from R^2 to R^3.

13. State Cayley-Hamilton theorem.

14. Obtain the quadratic form associated with the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & -2 \end{pmatrix}$.

PART – C
Essay

Answer **any four** questions from this Part. **Each** question carries **3** marks. **(4×3=12)**

15. If $v = (x^2 + y^2 + z^2)^{\frac{-1}{2}}$, Prove that $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0$.

16. If $\phi(cx - az, cy - bz) = 0$, show that $a\frac{\partial z}{\partial x} = b\frac{\partial z}{\partial y} = C$.

17. Evaluate $\int_0^{\pi} \frac{\sqrt{(1-\cos x)}}{1+\cos x} \sin^2 x dx$.

18. Evaluate $\int_0^a \frac{x^n}{\sqrt{(a^2 - x^2)}} dx$. Hence find the value of $\int_0^1 x^n \sin^{-1} x dx$.

19. Find a linearly independent eigenvectors of the matrix $\begin{pmatrix} 0 & 16 \\ 4 & 0 \end{pmatrix}$ and diagonalize it.

20. Prove that a square matrix A and its transpose A^T have the same characteristic roots.

PART – D
Long Essay

Answer **any two** questions from this Part. **Each** question carries **5** marks. **(2×5=10)**

21. If $x + y = 2e^{\theta}\cos\phi$ and $x - y = 2ie^{\theta}\sin\phi$, show that $\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy\frac{\partial^2 u}{\partial x \partial y}$.

22. Derive the formula for $\int_0^{\frac{\pi}{2}} \sin^p x \cos^q x dx$ where p and q are positive integers.

23. Give a set of three vectors in R^3 that are linearly dependent. Justify your answer.

24. Diagonalize the following matrix, if possible $A = \begin{pmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{pmatrix}$.