



K23U 0515

Reg. No. :

Name :

VI Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/
Improvement) Examination, April 2023
(2019 and 2020 Admissions)

CORE COURSE IN MATHEMATICS

6B12MAT : Numerical Methods, Fourier Series and Partial Differential
Equations

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **one** mark.

1. Forward difference operator $\Delta f(x_i) = \underline{\hspace{2cm}}$
2. Using Picard's method, obtain a solution up to the second approximation to the equation $\frac{dy}{dx} = 2y - x$ such that $y(0) = 1$.
3. Define odd function and give an example.
4. Define a periodic function and find the period of $\cos \pi x$.
5. Write the Laplacian equation in Polar coordinates.

PART – B

Answer **any 8** questions. **Each** question carries **two** marks.

6. Find the Lagrange interpolation polynomial for the following data :

| | | | |
|-------------|---|---|----|
| x | 1 | 2 | 4 |
| f(x) | 1 | 7 | 61 |

7. Find the second divided difference of $f(x) = \frac{1}{x}$, using points x_0, x_1, x_3 .
8. Show that $\mu = \sqrt{\left(1 + \frac{1}{4}\delta^2\right)}$.

P.T.O.



9. Obtain the approximate value of $y(1.2)$ for the initial value problem $y' = -2xy^2$, $y(1) = 1$ using Taylor series second order with step size $h = 0.1$.
10. Using Euler method, find $y(0.02)$, $y' = 2y$ with $y(0) = 1$ and $h = 0.01$.
11. Using Heun's method, find $y(0.2)$, $y' = x^2 + y^2$ with $y(0) = 1$ and $h = 0.1$.
12. State Euler formula for Fourier coefficients.
13. Find the Fourier series of $f(x) = x$, $-L < x < L$, $f(x + 2L) = f(x)$.
14. Verify that the function $u = x^2 + t^2$ is a solution of wave equation with suitable c .
15. Solve $u_{xx} - u = 0$.
16. Determine the type and normal form of the PDE $u_{xx} - 16u_{yy} = 0$.

PART - C

Answer **any four** questions. **Each** question carries **four** marks.

17. Find $\ln(9.2)$ with $n = 3$, using Lagrange interpolation formula with the given table :

| | | | | |
|-------------|---------|---------|---------|---------|
| x | 9.0 | 9.5 | 10 | 11.0 |
| ln x | 2.19722 | 2.25129 | 2.30259 | 2.39790 |

18. Using divided differences interpolation, find $f(x)$ as a polynomial if

| | | | | | | |
|-------------|----|----|----|---|---|----|
| x | -3 | -2 | -1 | 1 | 2 | 3 |
| f(x) | 18 | 12 | 8 | 6 | 8 | 12 |

19. Construct Newton's Backward Interpolation, table and Interpolating polynomial for the data :

| | | | | | | |
|-------------|------|-----|---|----|-----|-----|
| x | -4 | -2 | 0 | 2 | 4 | 6 |
| f(x) | -139 | -21 | 1 | 23 | 141 | 451 |

20. Using Picard's method, obtain a solution up to the fourth approximation to the equation $\frac{dy}{dx} = y + x$ such that $y(0) = 1$.



21. Given $\frac{dy}{dx} = 1 + y^2$ where $y(0) = 0$. Find $y(0.2)$ correct to four decimal places by Runge-Kutta second order formula.

22. Find the Fourier series of the function $f(x) = |x|$, $-2 \leq x \leq 2$ and $f(x+4) = f(x)$.

23. Consider the elastic string of length L whose ends are held fixed the string is set in motion from its equilibrium position with an initial velocity.

$$u_t(x, 0) = g(x) \begin{cases} \frac{2x}{L}, & 0 \leq x \leq \frac{L}{2} \\ \frac{2(L-x)}{L}, & \frac{L}{2} \leq x \leq L \end{cases}$$

Find the displacement $u(x, t)$ of the string.

PART - D

Answer **any two** questions. **Each** question carries **six** marks.

24. Using Lagrange interpolation, obtain the value of $e^{-0.15}$. Determine the maximum absolute error at this point. Compare it with actual error. If

| x | 0.1 | 0.2 | 0.4 |
|--------------|---------|---------|---------|
| $f(x) = e^x$ | .904837 | .818731 | .670320 |

25. Use Runge-Kutta fourth-order method with $h = 0.2$ to find the value of y at $x = 0.2$ and $x = 0.4$, given $\frac{dy}{dx} = 1 + y^2$ where $y(0) = 0$.

26. Find the Fourier series of the function $f(x) = \begin{cases} x, & -\frac{\pi}{2} < x \leq \frac{\pi}{2} \\ (\pi - x), & \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$.

27. Dérive D'alembert solution of wave equation.
