K22U 1293

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
2B02 MAT : Integral Calculus and Logic

Time : 3 Hours Max. Marks : 48

PART – A

Short answer questions. Answer **any 4**.

1. Find $\frac{d}{dx}\left(\sinh\left(e^{2x+3}\right)\right)$.
2. Convert the equation $(x-2)^2 + y^2 = 4$ into polar form.
3. Draw the domain of integration of the double integral $\int_1^2 \int_1^x f(x, y)\, dydx$.
4. Write the following statement using quantifiers and symbols :

   "There exists a number x such that for all y greater than 0, the sum of x and y is less than 0".
5. Write the contrapositive of the statement : $x + y \leq 0 \Rightarrow x^2 + y^2 > 0$. **(4×1=4)**

PART – B

Short essay questions. Answer **any 8**.

6. Show that $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x . \tanh x$.
7. Evaluate $\int_0^{\sqrt{\frac{\pi}{2}}} x\cos^5(x^2)dx$.
8. Evaluate $\int_0^1 \int_{x^2}^x (2xy - x^2)dydx$.
9. Write the equation of the infinite cone $z = \sqrt{x^2 + y^2}$ in spherical co-ordinates.

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10. Use double integration to find the area between two concentric circles of radius 2 and 4.

11. Find $\int_0^{\frac{\pi}{2}} \cos x \, dx$ using trapezoidal rule, taking two sub-intervals.

12. Find $\int_0^4 f(x)\,dx$ using Simpson's $\frac{1}{3}^{rd}$ rule, where the function f(x) is given by

x	0	1	2	3	4
f(x)	0	1	8	22	30

13. Explain the terms :
    i) Tautology
    ii) Disjunction of two statements.

14. State the two D'Morgan's laws for quantified statements.

15. Let P(x) be the statement : 'x is an even integer'. Check whether the statement 'P(3) $\Rightarrow$ P(5)' is true or false. Justify.

16. If a and b are positive real numbers with a > b, then prove that $\frac{1}{a} < \frac{1}{b}$. **(8×2=16)**

PART – C

Essay questions. Answer **any 4**.

17. Derive the reduction formula for $\int \sec^n x \, dx$, n > 2, and use it to evaluate $\int \sec^3 x \, dx$.

18. Find $\int \frac{dx}{\sqrt{9x^2 + 4x^4}}$ in terms of hyperbolic functions (assume x > 0).

19. Evaluate the integral $\int\int_R r^2 dr d\theta$, where R is the region lying outside the circle $r = \sqrt{2}$ and inside the cardioid $r = 2(1+\cos\theta)$.

20. Change the order of integration and then evaluate $\int_0^4 \int_{\frac{4-x}{2}}^{\frac{4+x}{2}} dy dx$.

21. Give an example of a non-constant function (along with domain) for which trapezoidal rule can give the exact value of definite integral. Justify your claim.

22. Prove the following statement by contrapositive method: "If n is an integer and n^2 is odd, then n must be odd". Is the converse true ?

23. Write the negation of the following statements using quantifiers:

    i) $(\forall x \in \mathbb{R})\ (2x^2 + x < 4)$

    ii) $(\exists x \in \mathbb{R})\ (2x^2 + x > 4)$. **(4×4=16)**

PART – D

Long essay questions. Answer **any 2**.

24. Evaluate the following definite integrals:

    i) $\int_1^2 \frac{\cosh(\ln t)}{2t}\, dt$

    ii) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{4 + \sin^2 x}}\, dx$.

25. Use triple integration to find the volume of the solid G that is bounded above by the paraboloid $z = x^2 + y^2 + 1$, below by the XY plane and laterally by the cylinder $x^2 + y^2 = 9$.

26. Find $\int_0^6 \frac{3}{x+1}\, dx$ using Simpson's $\frac{1}{3}^{rd}$ rule, taking six sub-intervals.

27. Prove that there are infinitely many prime numbers. **(2×6=12)**
