K25U 0160

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/ Supplementary/Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B12 MAT : Numerical Methods, Fourier Series and Partial Differential Equations

Time : 3 Hours Max. Marks : 48

I. Answer **any 4** out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1) Define first divided difference formula.

2) Define shift operator E.

3) Define explicit single step method to solve an ordinary differential equation.

4) Define the fundamental period.

5) Write one dimensional wave equation.

II. Answer **any 8** questions out of 11 questions. **Each** question carries **2** marks. **(8×2=16)**

6) Prove that $\Delta = E - 1$.

7) Prove that $\Delta\left(\frac{f_i}{g_i}\right) = \frac{g_i \Delta f_i - f_i \Delta g_i}{g_i g_i + 1}$.

8) Prove that $\Delta\left(f_i^2\right) = (f_i + f_{i+1})\Delta f_i$.

9) Describe quadratic interpolation.

10) State Existence Uniqueness theorem for an initial value problem.

P.T.O.

11) Explain Taylor series method.

12) If f(x) and g(x) have period p then prove that af(x) + bg(x) with any constant a and b also has the period p.

13) Define a periodic function with an example.

14) Solve $u_{xx} - u = 0$ like an ordinary differential equation.

15) State fundamental theorem on super position.

16) Verify that $u = \frac{y}{x}$ satisfies $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$ with $f(x, y) = \frac{2y}{x^3}$.

III. Answer **any 4** questions out of 7 questions. **Each** question carries 4 marks. **(4×4=16)**

17) Find the Lagrange interpolating polynomial at x = 3 for

x :	2.5	3.5
f(x) :	6	8

18) Prove that $E^{1/2}\,\delta = E - 1$.

19) Find the classical Runge-Kutta fourth order of $y' = x(y - x)$, $y(2) = 3$ in the interval [2, 2.4] with step size h = 0.2.

20) Obtain the approximate value of y(0.2) for the initial value problem $y' = x^2 + y^2$, $y(0) = 1$ with h = 0.1 using Euler method.

21) Find the Fourier coefficient of the periodic function $f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}$

and $f(x + 2\pi) = f(x)$. Hence prove that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$.

22) Find the d'Alembert's solution of the wave equation $u_{tt} - c^2 u_{xx} = 0$.

23) Find the type, transform to normal form and solve $u_{xx} + 4u_{yy} = 0$.

IV. Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

(2×6=12)

24) Prove that $\Delta + \nabla = \frac{\Delta}{\nabla} - \frac{\nabla}{\Delta}$.

25) Find the solution of the initial value problem $y' = 2y - x$, $y(0) = 1$ by performing three iteration of the Picard's method.

26) Find the even half range expansion of the function $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < \frac{L}{2} \\ \frac{2k}{L}(L - x) & \text{if } \frac{L}{2} < x < L \end{cases}$

27) Find the temperature u (x, t) in a laterally insulated copper bar 80cm long if the initial temperature is $100\sin\left(\frac{\pi x}{80}\right)^{\circ}$C and the ends are kept at 0°C. How long will it take for the maximum temperature in the bar to drop to 50°C ? Physical data for copper : density $8.92 g/cm^3$, specific heat 0.092cal/g°C, thermal conductivity : 0.095cal/(cmsec°C).