K21U 1532

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Sup./Imp.) Examination, November 2021
(2015 – '18 Admns.)
CORE COURSE IN MATHEMATICS
5B05MAT : Real Analysis

Time : 3 Hours Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** carries **1** mark.

1. Write the Supremum of $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$.
2. Define contractive sequences.
3. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.
4. State sequential criterion for continuity.

SECTION – B

Answer **any eight** questions. **Each** carries **2** marks.

5. Find all $x \in \mathbb{R}$ such that $\frac{2x+1}{x+2} < 1$.
6. If $x > -1$, show that $(1 + x)^n \geq 1 + nx \ \forall n \in \mathbb{N}$.
7. If $t > 0$, prove that there is an n_t in $\mathbb{N}$ such that $0 < \frac{1}{n_t} < t$.
8. Show that convergent sequences in $\mathbb{R}$ are bounded.
9. Suppose $X = (x_n)$, $Y = (y_n)$ and $Z = (z_n)$ are sequences in $\mathbb{R}$ such that $x_n \leq y_n \leq z_n \forall n \in \mathbb{N}$ and $\lim(x_n) = \lim(z_n)$. Show that $Y = (y_n)$ is convergent and $\lim(x_n) = \lim(y_n) = \lim(z_n)$.
10. Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$ converges.
11. Check the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$.

12. State and prove Abel's Lemma.

13. Let I be an interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f(I) is an interval.

14. If $f : I \to \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if (x_n) is a Cauchy sequence in A. Show that $(f(x_n))$ is also a Cauchy sequence in $\mathbb{R}$.

SECTION – C

Answer **any four** questions. **Each** carries **4** marks.

15. Show that the set $\mathbb{Q}$ of rational numbers is dense in the set $\mathbb{R}$ of real numbers.

16. For $a, b \in \mathbb{R}$, show that $|a + b| \leq |a| + |b|$ and deduce $\big||a| - |b|\big| \leq |a - b|$.

17. Let (x_n) be a sequence of real numbers such that $L = \lim\left(\frac{x_{n+1}}{x_n}\right)$ exists and let $L < 1$. Show that (x_n) converges and $\lim(x_n) = 0$.

18. State and prove the limit comparison test for series.

19. Let $Z = (z_n)$ be a decreasing sequence of strictly positive numbers with $\lim(z_n) = 0$. Show that the alternating series $\sum(-1)^{n+1}z_n$ is convergent.

20. Let $I = [a, b]$ be a closed bounded interval and let $f : I \to \mathbb{R}$ be continuous on I. Show that f has an absolute maximum and an absolute minimum on I.

SECTION – D

Answer **any two** questions. **Each** carries **6** marks.

21. a) State and prove nested intervals property.
    b) Using nested intervals property, show that [0, 1] is uncountable.

22. a) A sequence of real numbers is convergent if and only if it is a Cauchy sequence. Prove.
    b) Show that $\lim\left(n^{1/n}\right) = 1$.

23. a) State and prove Raabe's test.
    b) If a and b are positive numbers, show that $\sum(an + b)^{-p}$ converges if $p > 1$ and diverges if $p \leq 1$.

24. a) Let $I = [a, b]$ and let $f : I \to \mathbb{R}$ be continuous on I. If $f(a) < 0 < f(b)$, then there exists a number $c \in (a, b)$ such that $f(c) = 0$.
    b) Show that every polynomial of odd degree with real coefficients has at least one real root.
