K23P 1253

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Regular) Examination, October 2022
(2022 Admission)
STATISTICS WITH DATA ANALYTICS
MST1C03 : Distribution Theory

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries 2 marks.

1. For the p.m.f $P[X=j]=\frac{a_j\theta^j}{f(\theta)}, j=0,1,2...,\theta>0$, where $a_j \geq 0$ and $f(\theta)=\sum_{j=0}^{\infty} a_j\theta^j$, find the p.g. f of X.

2. Let $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$. Is X-Y a Poisson random variable with parameter $\lambda_1 - \lambda_2$. Justify your answer.

3. Define Cauchy distribution. If X has a Cauchy distribution C (1,0), what is the distribution of X^2.

4. Define a Bivariate distribution function.

5. If X_i, i = 1,2 ...n follow Exp (λ_i), what is the distribution of Min $(X_1, X_2, ...X_n)$.

6. The p.d.f. of a random variable X is given by $f(x) = ke^{-|x|}, -\infty<x<\infty$. Find k. Also obtain the characteristic function associated with X.

7. Define Central and Non-central F distribution.

8. What do you understand by "standard error" ? Give the expression for the standard error of the sample mean while sampling from the normal population $N(\mu, \sigma^2)$. **(8×2=16)**

P.T.O.

PART – B

Answer **any four** questions. **Each** question carries **4** marks.

9. Obtain the Poisson distribution as a limiting case of Negative Binomial distribution.

10. Derive the moment generating function of Normal distribution.

11. If X and Y are independent and distributed as G (α_1, β) and G (α_2, β). Derive the distribution of X+Y.

12. Derive the distribution of the range of a random sample of size n from U (0,1) distribution.

13. State and prove the lack of memory property of Exponential distribution.

14. Show that (X,Y) possesses a bivariate normal distribution if and only if every linear combination of X and Y, i.e., $aX + bY$, $a \neq 0$, $b \neq 0$, is a normal variate. **(4×4=16)**

PART – C

Answer **any four** questions. **Each** question carries **12** marks.

15. Define Power series distribution. Identify the members of the family. Also establish a recurrence relation satisfied by the cumulants of this family.

16. a) Define Geometric distribution. Find its moment generating function and hence find its mean and variance.

    b) If X_1, X_2 are i.i.d Geometric random variables, then show that the conditional distribution of X_1 given $X_1 + X_2$ is uniform.

17. If X_1, X_2 are independent rectangular variates on [0,1], find the distribution of

    i) X_1/X_2

    ii) X_1X_2

    iii) $X_1 + X_2$

    iv) $X_1 - X_2$

18. Define the non-central Chi-Square statistic and derive its p.d.f. Deduce the p.d.f of the central Chi-Square. State the additive property of then non-central Chi-Square.

19. Let $X_{(1)}, X_{(2)}, X_{(3)}$ be the order statistics of iid random variables X_1, X_2, X_3 having Exponential distribution with parameter β.

    i) Find the distribution of Let $X_{(1)}$ and $X_{(3)}$.

    ii) Show that $Y_1 = X_{(3)} - X_{(2)}$ and that $Y_2 = X_{(2)}$ are independent.

20. Let $X_1, X_2, ...X_n$ be a random sample of size n from $N(\mu, \sigma^2)$. Show that the sample mean $\bar{X}$ and sample mean S^2 are independently distributed. Deduce their distributions. **(4×12=48)**