K21U 4551

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **1** mark.

1. For $a, b \in \mathbb{R}$ if $a + b = 0$, then prove that $b = -a$.
2. Find the supremum of the set $\left\{1-\frac{1}{n} : n \in \mathbb{N}\right\}$.
3. Show that $\lim\limits_{n \to \infty} \frac{1}{n} = 0$.
4. Give an example of a discontinuous function on $\mathbb{R}$.
5. Define sequential criterion for the continuity of a function f on $\mathbb{R}$. **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

6. State and prove Archimedean property.
7. Determine the set $B = \left\{x \in \mathbb{R} : x^2 + x > 2\right\}$.
8. Let $J_n = \left(0, \frac{1}{n}\right)$ for $n \in \mathbb{N}$, prove that $\bigcap_{n=1}^{\infty} J_n = \varnothing$.
9. Prove that a sequence in $\mathbb{R}$ can have at most one limit.
10. Show that a convergent sequence of real numbers is bounded.
11. Prove that every convergent sequence is a Cauchy sequence.

P.T.O.

12. If the series $\sum x_n$ converges, then prove that $\lim_{n \to \infty} x_n = 0$.

13. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + n}$.

14. Show that every absolutely convergent series is convergent.

15. Show that the function $f(x) = \frac{1}{x}$ is not bounded on the interval $(0, \infty)$.

16. If functions f, g are continuous at a point c, then prove that f + g is also continuous at c. **(8×2=16)**

PART – C

Answer **any four** questions. **Each** question carries **4** marks.

17. Prove that the set of real numbers is not countable.

18. State and prove Squeeze theorem.

19. State and prove Bolzano Weierstrass theorem.

20. Let $X = (x_n)$ and $Y = (y_n)$ that converges to x and y respectively, then prove that X + Y converges to x + y.

21. Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

22. If $X = (x_n)$ is a convergent monotone sequence and the series $\sum y_n$ is convergent, then prove that the series $\sum x_n y_n$ is convergent.

23. State and prove preservation of intervals theorem. **(4×4=16)**

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that there exists a positive real number x such that $x^2 = 2$.

25. State and prove Monotone convergence theorem.

26. State and prove D'Alembert's ratio test for series.

27. If $f : [a, b] \to \mathbb{R}$ is a continuous function, then prove that f has an absolute maximum and absolute minimum on [a, b]. **(2×6=12)**