K22U 1059

II Semester B.Sc. Degree (C.B.C.S.S. – Supplementary)
Examination, April 2022
(2017-2018 Admissions)
CORE COURSE IN MATHEMATICS
2B02 MAT – Integral Calculus

Time : 3 Hours

Max. Marks : 48

SECTION – A

All the first 4 questions are compulsory. They carry 1 mark each :

1. State Fundamental theorem of Calculus.

2. Define Beta function.

3. Evaluate $\int_{-1}^{1} \int_{0}^{2} (1 - 6x^2 y) \, dx \, dy$.

4. Evaluate $\int_{0}^{\pi} 3\cos^2 x \sin x \, dx$.

SECTION – B

Answer **any 8** questions from among the questions **5 to 14.** These questions carry
2 marks each :

5. Suppose $\int_{1}^{x} f(t) \, dt = x^2 - 2x + 1$. Find $f(x)$.

6. Find the area of the region enclosed by the parabola $y = x^2 - 2$ and the line $y = 2$.

7. Evaluate $\int_{0}^{\infty} e^{-x^2} \, dx$.

8. Find the area of the region enclosed by the curves $y = \sin 2x$ & $y = 2\sin x$ in $0 \le x \le \pi$.

P.T.O.

9. Find $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

10. Calculate $\iint_R \frac{\sin x}{x} dA$, where R is the triangle in the xy plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

11. Find $\int_0^{\ln 2} 4e^x \sinh x \, dx$.

12. Show that if f is continuous on [a, b], $a \neq b$ and if $\int_a^b f(x)dx = 0$, then $f(x) = 0$ atleast once in [a, b].

13. Find the average value of $f(x) = x^2 - 1$ on $\left[0, \sqrt{3}\right]$.

14. Express $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}$ in terms of a Beta function.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4 marks each** :

15. Find the area of the region between the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \leq x \leq 2$ and the x – axis.

16. Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

17. Find the area of the surface generated by revolving the right hand loop of the lemniscate $r^2 = \cos 2\theta$ about the y – axis.

18. Show that $\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx = \beta(m, n)$, $m > 0$, $n > 0$.

19. Prove $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

20. Change the order of integration and evaluate $\int_0^1 \int_y^1 \sin(x^2)dx \, dy$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each** :

21. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.

22. Express $\int_0^1 x^m(1 - x^n)^p \, dx$ in terms of Beta function and hence evaluate

$$\int_0^1 x^5(1 - x^3)^3 \, dx .$$

23. The region in the first quadrant enclosed by the parabola $y = x^2$, the y – axis and the line $y = 1$ is revolved about the line $x = \dfrac{3}{2}$ to generate a solid. Find the volume of the solid.

24. Find the volume of the upper region D cut from the solid sphere $\rho \leq 1$ by the cone $\varphi = \dfrac{\pi}{3}$.

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