K21U 4552

Reg. No. : ....

Name : ....

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V Semester B.Sc. Degree CBCSS (OBE) Regular Examination, November 2021
(2019 Admn. Only)
CORE COURSE IN MATHEMATICS
5B07 MAT : Abstract Algebra

Time : 3 Hours Max. Marks : 48

PART – A

(Short Answer)

Answer **any 4** questions. **Each** question carries **1** mark.

1. Define abelian group with an example.
2. Is $\mathbb{Z}^*$ under division a binary operation. Justify.
3. Every infinite order cyclic group is isomorphic to
4. What is the order of alternating group A_n ?
5. State Lagrange's theorem. **(4×1=4)**

PART – B

(Short Essay)

Answer **any eight** questions. **Each** question carries **2** marks.

6. In a group G with binary operation $*$, prove that there is only one element e in G such that $e * x = x * e = x, \forall x \in G$.
7. Prove that $(\mathbb{Q}^+, *)$, where $*$ is defined by $a * b = \frac{ab}{2}$; $a, b \in \mathbb{Q}^+$ is a group.
8. For sets H and K, Let $H \cap K = \{x/x \in H \text{ and } x \in K\}$, show that if H and K are subgroups of a group G, then $H \cap K$ is also a subgroup of G.

P.T.O.

9. Prove that the order of an element of a finite group divides the order of group.

10. Explain the elements of group S_3.

11. Find the order of (14)(3578) in S_8.

12. Prove that every permutation σ of a finite set is a product of disjoint cycles.

13. Determine the permutation (18)(364)(57) in S_8 is odd or even.

14. State fundamental homomorphism theorem.

15. Find the order of $\mathbb{Z}_6/< 3 >$.

16. Let $\phi : G \rightarrow G'$ be a group homomorphism. Prove that Kerϕ is a subgroup of G.

(8×2=16)

PART – C

(Essay)

Answer **any four** questions. **Each** question carries 4 marks.

17. Prove that subgroup of a cyclic group is cyclic.

18. Let G be a group and $a \in G$. Prove that $H = \{a^n / n \in \mathbb{Z}\}$ is the smallest subgroup of G that contains a.

19. Determine whether the set of all $n \times n$ matrices with determinant -1 is a subgroup of G.

20. Let A be a non-empty set. Prove that S_A, the collection of all permutations of A is group under permutation multiplication.

21. Define rings. Prove that $(\mathbb{Z}_n, +_n, \times_n)$ is a ring.

22. Prove that every group is isomorphic to a group of permutations.

23. Prove that $\gamma : \mathbb{Z} \rightarrow \mathbb{Z}_n$; where $\gamma(m) = r$; where r is the remainder when m is divided by n is a homomorphism.

(4×4=16)

PART – D

(Long Essay)

Answer **any two** questions. **Each** question carries **6** marks.

24. Prove that every integral domain is a field.

25. Let H be a subgroup of G, then prove that the left coset multiplication is well defined by the equation (aH)(bH) = (abH) if and only if H is a normal subgroup of G.

26. a) Find all cosets of the subgroup < 2 > of $\mathbb{Z}_{12}$.

    b) Prove that every group of prime order is cyclic.

27. Let G be a cyclic group with n elements generated by a. Let b $\in$ G and b = a^s, then prove that b generates a cyclic subgroup H of G containing $\frac{n}{d}$ elements, where d is the gcd of n and s. **(2×6=12)**