K25U 0159

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (CBCSS – OBE – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B11MAT : Complex Analysis

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any 4** questions. **Each** question carries **1** mark.

1. Define a region in complex plane.
2. Evaluate $\oint_C e^z dz$ where C is the unit circle in counter clockwise direction.
3. State ML inequality for a complex function f(z).
4. What do you mean by singular point of a complex function ?
5. Define residue of a function f(z) at a singular point z_0. **(4×1=4)**

PART – B

Answer **any 8** questions. **Each** question carries **2** marks.

6. What do you mean by complex continuous function ? Give an example.
7. Show that $w = e^z$ is analytic everywhere.
8. Solve the equation $\cos z = 5$.
9. Evaluate $\oint_C \bar{z} dz$ where C is the unit circle in clockwise direction.

P.T.O.

10. Evaluate $\oint_C \frac{\cos z}{(z-\pi i)^2} dz$ where C is the circle $|z| = 4$ in counter clockwise direction.

11. Explain the idea of convergence of complex sequences. Give an example for a convergent complex sequence.

12. Prove that every absolutely convergent series is convergent.

13. Discuss the convergence of

$$\sum_{n=0}^{\infty} \frac{(100+75i)^n}{n!}.$$

14. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z-3i)^n.$$

15. State Laurent's theorem for a complex function f(z).

16. Prove that the zeros of a non-zero analytic function f(z) are isolated. **(8×2=16)**

PART – C

Answer **any 4** questions. **Each** question carries **4** marks.

17. Prove that an analytic function whose absolute value is a constant, is a constant function.

18. Prove that the function w = ln z analytic everywhere except at zero and on the negative real axis. Also find its derivative.

19. Evaluate $\int_C (z-z_0)^m dz$ where C is a circle with center at z_0 and radius r in counter clockwise direction.

20. If f(z) is analytic in a simply connected domain D, then prove that the integral of f(z) is independent of path in D.

21. Discuss the convergence of geometric series.

22. Find all the power series expansions of $\frac{1}{1-z}$ with center 0.

23. Integrate $f(z) = \frac{\sin z}{z^4}$ counter clockwise around the unit circle. **(4×4=16)**

PART – D

Answer **any 2** questions. **Each** question carries **6** marks.

24. State and prove the necessary condition for a function f(z) = u (x, y) + iv (x, y) to be analytic at a point z_0.

25. State and prove Cauchy's integral formula.

26. Find the Taylor's series for the function $f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$ at the point z = 1.

27. Find all Taylor and Laurent series of $f(z) = \frac{-2z + 3}{z^2 - 3z + 2}$ with center 0. **(2×6=12)**