M 5473

Reg. No. : ....

Name : ....

I Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal-Ul-Ulama Degree (CCSS – Regular/Supple./Improvement) Examination, November 2013

COMPLEMENTARY COURSE IN MATHEMATICS

1C01 MAT : Algebra and Geometry

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) ______________ is a group under multiplication but is not a group under addition.

   b) ______________ is an example of a 1-demensional vector space.

   c) ____________ is an example of a finite field.

   d) ______________ is an independent subset of $\mathbb{R}^2$. **(Weightage 1)**

Answer **any six** from the following : **(Weightage 1 each)**

2. Find the span of $\{(1, 0), (0, 1)\}$ in $\mathbb{R}^2$.

3. Prove or disprove that $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (x_1, x_2, 0)$ is a linear transformation.

4. Check whether the set of all $f \in \mathcal{C}[0, 1]$ such that $f(0) = f(1)$ is a subspace of $\mathcal{C}[0, 1]$

5. If (v_1, v_2) is a linearly independent set in V. Then prove that $v_1 \neq 0 \neq v_2$.

6. If $T : U \to V$ be a linear map. Then prove that

   $T(\alpha_1 u_1 + ... + \alpha_n u_n) = \alpha_1 T(u_1) + ... + \alpha_n T(u_n)$ where $u_1, u_2, ... u_n \in U$ and $\alpha_1, \alpha_2 ... \alpha_n$ are scalers.

P.T.O.

7. Explain the linear transformation "differential operator".

8. Define eigen vector of a matrix.

9. Write equations relating polar and cartesian co-ordinates.

10. Explain the representation of a point in spherical co-ordinates.

11. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

(Weightage 6×1=6)

Answer **any seven** from the following : **(Weightage 2 each)**

12. If S is a nonempty subset of a vector space V, then prove that [S] is the smallest subspace of V containing S.

13. If U and W are subspaces of a vector space V, prove that $U \cup W$ is a subspace of V iff $U \subset W$ or $W \subset U$.

14. If a set is linearly independent then prove that any subset of it is also linearly independent. What about the superset of a linearly independent set.

15. Find the linear transformation $T : V_2 \to V_2$ such that $T(1, 2) = (3, 0)$ and $T(2, 1) = (1, 2)$.

16. Find the rank of the matrix $\begin{bmatrix} 4 & 1 & 2 \\ -3 & 2 & 4 \\ 8 & -1 & -2 \end{bmatrix}$.

17. Show that the interchange of a pair of rows does not change the rank.

18. Verify Cayley Hamilton Theorem for the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$.

19. Investigate the values of λ and μ so that the equations $2x+3y+5z=9,\ 7x+3y-2z=8\ ,\ 2x+3y+\lambda z=\mu$ have a unique solution.

20. Solve the system of equations.

$x+y+z=9,\ 2x+5y+7z=52,\ 2x+y-z=0.$

21. Show that the equations $x_1+3x_2-x_3=1,\ 4x_1+15x_2-5x_3=7,$ $3x_1+3x_2-x_3=0$ are consistent and then solve.

22. Show that A and its transpose has the same eigen values. **(7×2=14)**

(Answer **any three** from the following : **(Weightage 3 each)** :

23. Find the eigen values and the corresponding eigen vectors of the matrix.

$$\begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

24. Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$ and evaluate A^{-1}.

25. 1) Convert the polar equation $r\cos\theta=2$ into cartesian equations.

2) Convert the cartesian equation $\frac{x^2}{9}+\frac{y^2}{4}=1$ into polar equations.

26. Translate the equation $x^2+y^2+(z-1)^2=1,\ z\le 1$ into cylindrical and spherical system.

(Weightage 3×3=9)

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