M 961

Reg. No. : ....

Name : ....

II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Reg./Supple./Improv.) Examination, April 2012

CORE COURSE IN MATHEMATICS

2B02 MAT : Foundations of Higher Mathematics

Time: 3 Hours

Max. Wightage : 30

1. Fill in the blanks :

   a) $1 + 2x + 3x^2 + 4x^3 + ... =$ ....

   b) The number of terms in the expansion of $(1 - x)^{-1} =$ ....

   c) The n-th term of the series $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + ... =$ ....

   d) $\lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n =$ ....

(Weightage 1)

2. Fill in the blanks :

   a) The dual of $A \cup (A' \cap B) = A \cup B$ is ....

   b) Consider the relation defined by $y^2 = 16x$, then the graph of the equation is ....

   c) If R is the relation in the natural numbers $N = \{1, 2, 3, ...\}$ defined by the open sentence "x divides y", then R = ...

   d) If $A = \{a, b\}$, $B = \{2, 3\}$ and $C = \{3, 4\}$, then $(A \times B) \cap (A \times C) =$ ....

(Weightage 1)

Answer **any five** from the following :

(Weightage 1 each)

3. Sum the series $1 + \frac{1}{5} + \frac{1.4}{5.10} + \frac{1.4.7}{5.10.15} + ...$

4. Sum the series $\frac{1}{1.3} + \frac{1}{2.5} + \frac{1}{3.7} + ...$

P.T.O.

5. If $A \cup B = U$, prove that $A' \subset B$.
6. Find all the partitions of $S = \{1, 2, 3, 4\}$.
7. If R is a relation defined on the set of natural numbers and R is given by (a, b) is related to (c, d) if and only a + d = b + c, prove that R is an equivalence relation.
8. If A, B, C are any three sets such that ACB and CCD, then prove that $(A \times C) \subset (B \times D)$.
9. If the relation in N defined by 'x divides y' is a partial order, the insert the correct symbol <, > or || between each pair of numbers.

   a) 3 18

   b) 16 26

   c) 8 2

   d) 5 .... 20.
10. Define lattice. **(5×1=5)**

Answer **any seven** from the following : **(Weightage 2 each)**

11. Each of the following open sentences defines a relation R in the natural number N. State whether or not each relation is transitive.

    a) x is less than or equal to y.

    b) x divides y,

    c) x + y = 10.
12. If the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$, find gof and fog.
13. If $f : A \to B$ and $g : B \to C$ prove the following :

    a) If gof is one-to-one, then f is one-to-one

    b) If gof is onto, then g is onto.
14. a) If $f : A \to B$, $g : B \to C$ and $h : C \to D$ are functions, prove that ho(gof) = (hog)of.

    b) Can a constant function be an onto function ?
15. If $f : A \to B$ is an onto function and $g : B \to C$ is also onto, prove that gof is also onto.
16. If A = {2, 3, 4, ...} is ordered by "x divides y", then find

    a) All minimal elements and

    b) All maximal elements.

17. If L is a finite complemented distributive lattice then show that every element a in L is a join of a unique set of atoms.
18. Transform the equation $25x^4 + 5x^3 - 7x^2 + 1 = 0$ into another with integral co-efficients and the leading co-efficient unity.
19. Solve the equation $x^4 + x^3 - 33x^2 + 61x - 14 = 0$, given that $2 + \sqrt{3}$ is a root.
20. If α, β, γ are the roots of the equation $x^3 + px^2 + qx = 0$, find the value of

a) $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\alpha\gamma}$ and

b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. **(Weightage 7×2=14)**

Answer **any three** from the following : **(Weightage 3 each)**

21. If α, β, γ are the roots of $x^3 + qx + r = 0$ find the equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha)^2, (\alpha - \beta)^2$.
22. Show that $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \ldots = 27e$.
23. Show that $\frac{1}{2.3.4} + \frac{1}{4.5.6} + \frac{1}{6.7.8} + \ldots . = \frac{3}{4} - \log 2$.
24. Find the sum of the fourth powers of the roots of the equation $x^4 - 5x^3 + x - 1 = 0$.
25. Solve by Gardan's method : $x^3 - 9x + 28 = 0$. **(Weightage 3×3=9)**
