M 975

Reg. No. : ....

Name : ....

II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Reg./Supple./Improv.) Examination, April 2012

COMPLEMENTARY COURSE IN STATISTICS (For Maths/Computer Science Core)

2C02 STA : Probability Theory and Random Variables

Time: 3 Hours Max. Weightage : 30

Instruction : ***Use** of calculators and tables are **permitted.***

PART – A

Answer **any 10** questions (Weightage **1 each**) :

1. Define with suitable examples,
   a) Sample space
   b) Mutually exclusive events.
2. State the mathematical definition of probability.
3. If A, B and C are the possible events in a random experiment, write down the set theoretic equivalent of the following :
   a) None of the three events occur
   b) Exactly one of the three events occurs
   c) At least one of the events occurs.
4. What are the axioms of probability ?
5. Given $P(A) = 0.5$, $P(A \cup B) = 0.7$. Find $P(B)$ if $P(A/B) = 0.5$.
6. Define independence of two events.
7. A fair six sided die is rolled twice. What is the conditional probability that both faces will show even numbers given that the sum of the faces is eight.
8. Distinguish between discrete and continuous random variables.

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9. What are the properties of a probability density function ?
10. Define distribution function of a random variable.
11. Define a joint probability density function. **(10×1=10)**

PART – B

Answer **any 6** questions (Weightage **2 each**) :

12. State and prove addition law of probability.
13. If A and B are any two events in a sample space S show that
    a) $P(A \cap B) \geq P(A) + P(B) - 1$ and
    b) $P(A \cup B) \leq P(A) + P(B)$.
14. State and prove the multiplication law of probability.
15. Probabilities that a husband and wife will be alive 20 years from now is given by 0.8 and 0.9 respectively. Find the probability that in 20 years :
    i) both
    ii) neither
    iii) at least one will be alive.
16. For any three events A, B and C show that
    $P[(A \cup B)/C] = P(A/C) + P(B/C) - P[(A \cap B)/C]$.
17. Examine the consistency of the following information
    $P(A) = 0.3$, $P(B) = 0.2$, $P(C) = 0.8$, $P(AB) = 0.2$, $P(BC) = 0.4$, $P(AC) = 0.2$,
    $P(ABC) = 0.1$ and $P(A'B'C') = 0.15$.
18. If $f(x) = K \cdot \frac{1}{1+x^2}$, $-\infty \leq x \leq \infty$ represents a probability density function , evaluate the constant K and determine the distribution function.
19. For a random variable X, the probability density functions
    $f(x) = \frac{x}{2}$ when $0 \leq x \leq 2$
    $= 0$ otherwise. Find the value of a if $P\left(X < a / X > a/2\right) = 1/2$.
20. Let X be a uniform random variable in the interval (0, 1) with the probability density function $f(x) = 1, 0 \leq x \leq 1$ obtain the probability density function of $y = -2 \log x$.

(6×2=12)

PART – C

Answer **any two** questions : **(Weightage : 4 each)**

21. State and prove Baye's theorem and indicate its importance. What are the objections against the use of Baye's theorem ?

22. A continuous random variable has the following p.d.f. :

$$f(x) = x \quad \text{if} \quad 0 < x \leq 1$$
$$= 2 - x \quad \text{if} \quad 1 \leq x \leq 2$$
$$= 0 \quad \text{otherwise.}$$

Evaluate the distribution function and calculate $P(0.3 < X \leq 1.5)$.

23. If $f(x, y) = \frac{2x + 3y}{72}, x = 0, 1, 2.\ y = 1, 2, 3$ is the joint p.d.f. of (X, Y) find,
    i) The distribution of $X + Y$
    ii) Conditional distribution of X given $X + Y = 3$
    iii) Examine whether X and Y are independent.

24. Given $f(x, y) = k\,xy,\ 0 < x < y < 1$

$$= 0 \quad \text{elsewhere. Find,}$$

   i) the value of the constant k.
   ii) the marginal density functions of X and Y
   iii) the conditional distribution of X given Y. **(2×4=8)**