M 10251

Reg. No. : ....

Name : ....

II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. (CCSS – Regular/Supplementary/Improvement) Degree Examination, March 2011
MATHEMATICS (Core Course)
2B02 MAT : Foundation of Higher Mathematics

Time: 3 Hours Max. Weightage : 30

1. Fill the blanks :

a) Sum of the series $2\left[1+\frac{1}{3!}+\frac{1}{5!}+....\right]=$ ____________

b) For $|x|<1$, $1-x+x^2-x^3+....=$ ____________

c) Coefficient of x^n in the expansion of $1+\frac{1+2x}{1!}+\frac{(1+2x)^2}{2!}+\frac{(1+2x)^3}{3!}+...$ to ∞ is ____________

d) $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n=$ ________ **(Weightage 1)**

2. a) The number of relations from set $A=\{a, b, c\}$ to set $B=\{1, 2, 3\}$ is ________

b) A partition of set $A=\{x, y, z\}$ is ____________

c) $f(x)=x^2$, $g(x)=x+3$ then $(g\circ f)(2)=$ ____________

d) Domain of $f(x)=\sqrt{25-x^2}$ is ____________ **(Weightage 1)**

Answer **any five** from the following (Weightage **1 each**) :

3. Sum the series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+....$

4. Sum the series $\left(1+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)\frac{1}{9}+\left(\frac{1}{5}+\frac{1}{6}\right)\frac{1}{9^2}+......$

P.T.O.

5. Prove $(A \times B) \cap (A \times C) = A \times (B \cap C)$.

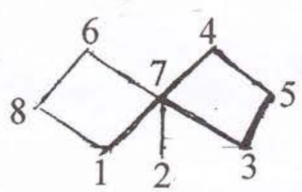
6. $R = \{(1,2), (1,3), (3,1), (3,3), (2,3)\}$ is a relation as $A = \{1, 2, 3\}$ find $R \circ R$.

7. Sketch the graph $f(x) = x^2 + x + 1$.

8. Let $f : R \to R$ be defined by $f(x) = \frac{2}{3}x + \frac{4}{5}$ find the formula for $f^{-1}(x)$.

9. Define a lattice.

10. Consider the ordered set A in Fig. 1



Find all minimal and maximal elements of A. **(Weightage 5×1=5)**

Answer **any seven** from the following (Weightage **2 each**) :

11. Let Z be the set of integers. For $x, y \in Z$ define $xRy \cdot f\ x \equiv y \pmod 5$ which means $x - y$ is divisible by 5. Prove R is an equivalence relation. Find all its equivalence classes.

12. Given $A = \{1, 2, 3, 4\}$ $B = \{u, v, w)$. Let R be the following relation from A to B : $R = (1, v), (1, w), (3, v), (4, u), (4, w)\}$
    a) Determine the matrix of the relation
    b) Draw the arrow diagram of R
    c) Find the inverse relation R^{-1} of R
    d) Determine the range of R and R^{-1}.

13. Prove that the function $f : A \to B$ is invertible if and only if f is bijective.

14. Sketch the relation in the plane $x^2 + 4y^2 \leq 16$. Find the Domain of this relation.

15. Suppose A and B are finite sets with | A | elements and | B | elements respectively. Prove that there are $|B|^{|A|}$ functions from A into B.

16. Define a partial order relation on a set S. When we say S is linearly ordered ? Give an example of a set with a partial order which is not linearly ordered.

17. Let L be a lattice. Then prove that complements are unique if they exist.

18. If α, β, γ be the roots of the Cubic $x^3 + px^2 + qx + r = 0$ find the value of $(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$.

19. Find the equation whose roots are the roots of the equation $5x^3 - 13x^2 - 12x + 7 = 0$ each diminished by 23.

20. Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$ whose roots are in arithmetical progression.

(Weightage 7×2=14)

Answer **any three** from the following (Weightage **3 each**) :

21. Sum to infinity the series $\frac{1}{1.2.3} + \frac{1}{5.6.7} + \frac{1}{9.10.11} + ...$

22. Sum to infinity the series $\sum_{n=1}^{\infty} \frac{n^2}{n+2} \frac{x^n}{n!}$.

23. a) Let L be a bounded distributive lattice then prove the complements are unique if they exist.

b) Give an example of a nonlinearly ordered set where inf (a, b) and sup (a, b) do exist for every a, b in the set.

24. Solve the equation $x^3 - 21x - 344 = 0$ by Cardan's method.

25. If $\alpha+\beta+\gamma=1, \alpha^2+\beta^2+\gamma^2=2, \alpha^3+\beta^3+\gamma^3=3$ form the equation whose roots are α, β, γ. Hence show that $\alpha^4+\beta^4+\gamma^4=\frac{25}{6}$.

(Weightage 3×3=9)