M 10263

Reg. No. : ....

Name : ....

II Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. (CCSS-Regular/Supplementary/Improvement) Degree Examination, March 2011
STATISTICS (Complementary Course for Maths/Comp. Sci. Core)
2C02 STA : Probability Theory and Random Variables

Time : 3 Hours Total Weightage : 30

***Instruction** : Use of calculators and statistical tables **permitted**.*

PART – A

Answer **any 10** questions. Weightage **1 each.**

1. Define statistical event. Give one example.
2. Define discrete sample space. Give an example with infinite number of sample points.
3. Mention two important drawbacks of classical definition of probability.
4. What is the probability that a normal year selected at random contains 53 Sundays ?
5. State the multiplication rule for two events.
6. If A and B are two independent events, prove that A and $\overline{B}$ are independent.
7. Define partitioning of a sample space.
8. Define the distribution function of a random variable.
9. If X is a continuous random variable and g (X) is an increasing or decreasing functions of X, write down the formula for the density function of g (X).
10. The joint density function of two random variables X and Y is given as follows :

$$f(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal density of Y.

P.T.O.

11. Two random variables X and Y have the joint mass function

$$f(x, y) = \frac{2x + y}{27};\ \begin{array}{l} x = 0, 1, 2 \\ y = 0, 1, 2 \end{array}$$

Find P {X = 0}.

PART – B

Answer **any 6** questions. Weightage **2 each.**

12. Explain the axiomatic definition of probability.

13. State and prove the addition rule for two events.

14. A problem is given to three students whose chances of solving it are $^1/_3$, $^1/_2$ and $^3/_4$. What is the probability that the problem is solved, if they try independently ?

15. If P (A) = 0.3, P (B) = 0.2 and P (A $\cap$ B) = 0.1, find the probability that :
    1) Exactly one of the events will happen
    2) At least one of the events will happen
    3) None of the events happen.

16. A factory produces a certain type of output by machines I, II and III. The daily production figures of these machines are 30%, 25% and 45% respectively. It is known that 1%, 1.2% and 2% of the outputs respectively of these machines are defective. An item drawn from a day's production is found to be defective. What is the probability that it came from machine II ?

17. A random variable X has the following density function

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } 0 \le x < 1 \\ \frac{1}{2} & \text{if } 1 \le x < 2 \\ \frac{1}{2}(3 - x) & \text{if } 2 \le x < 3 \end{cases}$$

Find the distribution function of X.

18. If F (x) is the distribution of a continuous random variable x, find the density function of Y = F (X).

19. Two random variables X and Y have the following joint density function :

$$f(x, y) = \begin{cases} K(4 - x - y) & \text{if } 0 \le x \le 2, 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

Find :

1) The value of K

2) The marginal density functions

3) Conditional density functions.

20. A two dimensional random variable (X, Y) has the following density function :

$$f(x, y) = \frac{x + y}{18}; \begin{matrix} x = 0, 1, 2 \\ y = 0, 1, 2 \end{matrix}$$

Find the marginal distributions of X and Y. Also find :

1) The conditional distribution of X when Y = 0

2) The conditional distribution of Y when X = 1.

PART – C

Answer **any 2** questions. Weightage **4 each**.

21. State and prove Bayes theorem. The probabilities that A, B, C are appointed as managers of a company are in the ratio 4 : 2 : 3. The probabilities that bonus scheme will be introduced if A, B, C becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively. What is the probability that bonus scheme will be introduced in the company ?

22. A random variable X has the density function :

$f(x) = Ae^{-x/5}$; $x > 0$

1) Find the value of A.
2) For any two numbers s and t, show that

$P\{X > s + t / X > s\} = P\{X > t\}$

3) Find the distribution function of X.

23. A random variable X has the following distribution :

Value of X :	0	1	2	3	4	5	6	7	8
Probability :	k	3k	5k	7k	9k	11k	13k	15k	17k

1) Determine the value of k.
2) Find $P\{X < 3\}$, $P\{X \geq 3\}$, $P\{0 < X < 5\}$.
3) What is the smallest value of x for which $P\{X \leq x\} > 0.5$?
4) Find the distribution function of X.

24. Two random variables X and Y have the following joint density function :

$$f(x, y) = \begin{cases} k(6 - x - y) & \text{if } 0 < x < 2, \\ & \quad 2 < y < 4 \\ 0 & \text{otherwise} \end{cases}$$

1) Find the value of k.
2) Find $P\{X < 1 \cap Y < 3\}$
3) Find $P\{X + Y < 3\}$
4) Find $P\{X < 1 \mid Y < 3\}$.
