M 6574

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
CORE COURSE IN MATHEMATICS
2B02 MAT : Foundations of Higher Mathematics

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

 a) The number of terms in the expansion of $(1-x)^{-1}$ is ____________

 b) $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots=$ ____________

 c) The n^{th} term of the series $\frac{2.3}{3!}+\frac{3.5}{4!}+\frac{4.7}{5!}+\ldots$ is ____________

 d) $\lim\limits_{n\to\infty}\left(1-\frac{1}{n}\right)^n=$ ____________ **(Weightage : 1)**

2. Fill in the blanks :

 a) The dual of $(A\cap U)\cap(\phi\cup A')=\phi$ is ____________

 b) Consider the relation defined by $x^2+y^2=16$, then graph of the equation is a ____________

 c) If $A=\{1, 2\}$, $B=\{a, b, c\}$ and $c=\{c, d\}$, then $(A\times B)\cap(A\times C)$ is ________

 d) If $R=\{(x, y)/x\in\mathbb{R}, y\in\mathbb{R}, 4x^2+9y^2=36\}$, then $R^{-1}=$ ________ **(Weightage : 1)**

Answer **any five** from the following (Weightage **1 each**) :

3. Sum the series $1+\frac{1}{4}+\frac{1.4}{4.8}+\frac{1.4.7}{4.8.12}+\ldots$

4. Prove that $\log 2-\frac{(\log 2)^2}{2!}+\frac{(\log 3)^3}{3!}=\ldots=\frac{1}{2}$.

P.T.O.

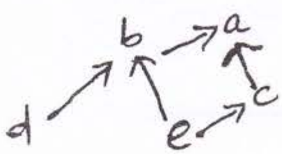
5. Prove that $(B \cap C) \cup A = (B \cup A) \cap (C \cup A)$.

6. Find all partitions of S= {1, 2, 3}.

7. If ~ is a relation on the set of natural numbers defined by (a, b) ~ (c, d) if and only ad = bc, then prove that '~' is an equivalence relation.

8. If R = {(1, 2), (2, 3), (3, 3)} is a relation defined on a set A, find R^2 and R^3.

9. If V= {a, b, c, d) is ordered by the following diagram, insert the correct symbol <, > or II between each pair of elements.



i) a .... e

ii) b .... e

iii) d ....a

iv) c .... d

10. If R is a relation defined on the set of natural numbers given by

$R = \{(x, y)/x \in N, y \in N, 2x + y = 10\}$, find

i) the domain of R

ii) the range of R

iii) R^{-1}. **(Weightage 5×1=5)**

Answer **any seven** from the following (Weightage **2 each**) :

11. If a relation R is transitive prove that its inverse is also transitive.

12. If f and g are function defined on the real numbers given by

$f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$, find $f \circ g$ and $g \circ f$.

13. If $f : A \to B$ is one-to-one and $g : B \to C$ is also one-to-one, prove that $g \circ f : A \to C$ is one-to-one.

14. If $f : A \to B$ and $g : B \to C$ have inverse functions $f^{-1} : B \to A$ and $g^{-1} : C \to B$, show that $g \circ f$ has an inverse function which is $f^{-1} \circ g^{-1} : C \to A$.

15. Prove that $f : A \to B$ is invertible if and only if f is bijective.

16. If f is a one-to-one and onto function defined on real numbers by $f(x) = 2x - 3$, find a formula that defines the inverse function f^{-1}.

17. If $B = \{2, 3, 4, 5, 6, 8, 9, 10\}$ is ordered by "x is a multiple of y". Find

    a) all maximal elements of B,

    b) all minimal elements of B and

    c) does B have a first or last element ?

18. If L is a complemented lattice with unique complements, then show that the join of irreducible elements of L other than zero are its atoms.

19. If α, β, γ are the roots of the equation $x^2 + ax - b = 0$, find the value of

    $$\frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\gamma}{\alpha\beta}.$$

20. Transform the equation $25x^4 + 5x^3 - 7x^2 + 1 = 0$ into another with integral co-efficients and the leading co-efficient unity. **(Weightage 7×2=14)**

Answer **any three** questions from the following (Weightage **3 each**) :

21. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, find the equation whose roots are

    $$\alpha + \frac{1}{\beta\gamma}, \beta + \frac{1}{\gamma\alpha}, \gamma + \frac{1}{\alpha\beta}.$$

22. Prove that $\sum_{n=0}^{\infty} \frac{5n+1}{(2n+1)!} = \frac{e}{2} + \frac{2}{e}$.

23. Show that $\frac{1}{2.3.4} + \frac{1}{4.5.6} + \frac{1}{6.7.8} + \ldots = \frac{3}{4} - \log 2$.

24. Solve the equation $4x^3 - 24x^2 + 23x + 18 = 0$ whose roots are in A.P.

25. Solve by the equation $x^3 - 9x + 28 = 0$ by Cardan's method. **(Weightage : 3×3=9)**