M 8749

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2015
(2013 and Earlier Adm.)
Complementary Course in Statistics
(For Maths./Comp. Sci. Core)
2C02 STA : PROBABILITY THEORY AND RANDOM VARIABLES

Time : 3 Hours Max. Weightage : 30

***Instruction** : **Use** calculators and tables are **permitted**.*

PART – A

Answer **any 10** questions. Weightage **1**.

1. Define the following terms :

   a) Random experiment

   b) Equally likely events

   c) Statistical regularity

   d) Probability space

2. State the relative frequency definition of probability. What are its advantages ?

3. State the axioms of probability measure.

4. What is the probability of getting a sum of 10 or more while throwing two dice ?

5. Distinguish between disjoint events and independent events. Can two disjoint events be independent ?

6. State addition theorem of probability for any three events A, B and C.

7. If A and B are any two events in a sample space S show that $P(A \cap B)' = P(A) - P(A \cap B)$.

8. Define a discrete random variable. Give an example.

P.T.O.

9. What are the properties of a probability density function ?

10. For what value of k does the function $f(x) = k.c^{-x/2}$, $x \geq 0$ represent a probability density function ?

11. Define a joint distribution function. **(10×1=10)**

PART – B

Answer **any 6** questions. Weightage **2.**

12. If A, B and C are any three events, prove that

    1) $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$

    2) $P(A \cap B \cap C) \geq P(A) + P(B) + P(C) - 2$.

13. What is the probability of having 53 Sundays in a leap year ?

14. Two urns each contains balls of different colours as stated below :

    Urn I : 4 black 3 red 3 green

    Urn II : 3 black 6 red 1 green

    An urn is chosen at random and two balls are drawn from it. What is the probability that one is green and the other is red ?

15. Define conditional probability. Show that it satisfies the three axioms of probability.

16. If $A_1, A_2, A_3, \dots A_n$ are n independent events with probabilities of occurrence $P_1, P_2, \dots, P_n$ show that $P(A_1 \cup A_2 \cup \dots A_n) = 1 - (1 - P_1), (1 - P_2) \dots (1 - P_n)$.

17. If A is independent of B, show that (i) A is independent of B' and (ii) A' is independent of B'.

18. Define distribution function of a random variable. State its important properties.

19. For a random variable X, the probability density function is

$$f(x) = \frac{x}{2} \text{ when } 0 \le x \le 2$$
$$= 0 \text{ elsewhere}$$

Find the value of 'a' if $P\left[x < a / x > a/2\right] = 1/2$.

20. If X has the p.d.f. $f(x) = \frac{1}{\sqrt{2\pi}} \cdot e^{-x^2/2}$, $-\infty \le x \le \infty$ find the density function of $y = X^2$. **(2×6=12)**

PART – C

Answer **any two** questions. Weightage **4 each**.

21. A random variable X has the following probability mass function.

X	0	1	2	3	4	5	6	7
f (x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find :

a) the value of K

b) $P(X \le 5)$

c) $P(X > 4)$

d) the smallest value of x for which $P(X \le x) > 1/2$.

22. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} ax & \text{if } 0 \le x \le 1 \\ a & \text{if } 1 \le x \le 2 \\ -ax + 3a & \text{if } 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

Determine :

i) the constant a

ii) $P(X \le 1.5)$.

23. Let (X, Y) be a pair of random variable with the joint p.d.f.

$f(x, y) = K(1 + x + y)^{-n}, x > 0, y > 0, n > 2$

$= 0$ elsewhere

Find the value of K and examine whether X and Y are independent random variables.

24. State and prove Baye's theorem. An insurance company insured 1400 car drivers, 3600 bus drivers and 5000 truck drivers. The probability of an accident is 0.06 for car driver, 0.02 for bus driver and 0.1 for truck driver. One of the insured persons meets with an accident, what is the probability that he is a bus driver ? **(2×4=8)**