K15U 0278

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree (CCSS-2014 Admn.-Regular) Examination, November 2015
Core Course in Mathematics
3B03 MAT : ELEMENTS OF MATHEMATICS – I

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Determine the number of different injections from {1, 2} into {a, b, c}.
2. Form an equation whose roots are 3 times those of the equation $2x^3 - 5x^2 + 7 = 0$.
3. State the fundamental theorem of algebra.
4. Find the number of solutions to the Diophantine equation, $2x + 10y = 17$.

(4×1=4)

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each.**

5. Show that $p \wedge q$ logically implies $p \leftrightarrow q$.
6. Determine the truth value of the statement $\forall x \forall y, x^2 + y^2 < 12$, where U = {1, 2, 3} is the universal set.
7. Form a rational cubic equation which shall have for roots $1, 3 - \sqrt{-2}$.
8. If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 + ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \alpha^2 \beta \gamma$.

P.T.O.

9. Find the sum of the cubes of the roots of the equation, $x^5 = x^2 + x + 1$.
10. Determine completely the nature of the roots of the equation, $x^5 - 6x^2 - 4x + 5 = 0$.
11. Find the sum of the trigonometric series, $\sin x + \sin 2x + \sin 3x + \ldots$.
12. Find the remainder when the sum, $1! + 2! + 3! + 4! + \ldots + 99! + 100!$ is divided by 12.
13. Show that there are infinitely many primes.
14. Show that the number $\sqrt{2}$ is irrational. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**.

15. a) What is an argument ? When do you say that an argument is valid ?

    b) Show by example that the validity of an argument does not depend upon the truth values nor the content of the statements appearing in the argument, but upon the particular form of the argument.
16. Solve the equation $81x^3 - 18x^2 - 36x + 8 = 0$ whose roots are in harmonic progression.
17. Solve the equation, $60x^4 = 736x^3 + 1433x^2 - 736x + 60 = 0$.
18. Solve, $x^4 - 3x^2 - 6x - 2 = 0$.
19. Let a and b be integers, not both zero. Show that a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.
20. Using Euclidean algorithm, find gcd (12378, 3054). **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. a) If S is a countable set, show that there exists a surjection of $\mathbb{N}$ onto S.

    b) If A is any set, show that there is no surjection of A onto the set P(A) of all subsets of A.

    c) Show that the collection P($\mathbb{N}$) of all subsets of the natural numbers $\mathbb{N}$ is uncountable.

22. Find the equation whose roots are the squares of the differences of roots of the equation $x^3+qx+r=0$.

23. a) Solve $x^4 - 9x^2 + 4x + 12 = 0$, given that the equation has multiple roots.

    b) Find the condition that all the roots of the equation $x^3 + px + q = 0$ may be real.

24. State and prove the Fundamental Theorem of Arithmetic. **(2×6=12)**

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