

Examination, May 2013 COMPLEMENTARY COURSE IN STATISTICS FOR MATHS/ COMPUTER SCIENCE CORE 4C04 STA : Statistical Inference

Time: 3 Hours

Max. Weightage: 30

Instruction : Use of scientific calculator permitted statistical tables are permitted.

PART-A

Answer any 10 questions.

(Weightage 1 each)

- 1. Define standard errors.
- 2. A sample of size 16 is taken from a normal population with mean 50 and standard deviation 20. What is the probability that the sample mean is at least 60 ?
- 3. Find the mean of Chi-square distribution with n degrees of freedom.
- 4. Write down the density function of student's t-distribution with n degrees of freedom. Hence find its mean.
- 5. Define F-statistic. Write down its probability density function.
- 6. Define unbiased estimator.
- 7. State Neyman-Pearson theorem.
- 8. Define Type I and Type II errors.
- 9. Define minimum variance unbiased estimator.
- 10. Distinguish between parametric and non-parametric tests.
- 11. Write down the expression for x^2 for testing the independence of attributes in a 2×2 contingency table. (10×1=10)

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PART-B

Answer any 6 questions :

(Weightage 2 each)

- 12. Derive the variance of t-distribution.
- 13. If F has F-distribution with (n_1, n_2) degrees of freedom, show that $\frac{1}{F}$ has

F-distribution with (n_2, n_1) degrees of freedom.

- 14. If $x_1 x_2 \dots x_n$ is a sample of size n from a Bernoulli population with parameter p, find unbiased estimators of p and p^2 .
- 15. If Tn is an estimator of a parameter θ such that E Tn = θ and V (Tn) $\rightarrow 0$ as $n \rightarrow \infty$, show that Tn is consistent for θ .
- Obtain a 100 (1 − 2)% confidence interval for the mean of a normal population when the population variance is (1) unknown (2) known.
- 17. Let $x_1 x_2 \dots x_n$ be a sample of size n from a population with density function $f(x) = \begin{cases} \frac{1}{\theta} & \text{if } 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$

Find the MLE of θ .

- 18. To test $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$ where p is the probability of getting a head in a single toss of a coin, the coin is tossed 5 times. It is decided to reject H_0 if more than 3 heads are obtained. Find the probability of Type I error and power of the test.
- 19. Obtain the most powerful size α test for $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (\theta_1 > \theta_0)$ for the population density

 $f(x) = \begin{cases} \theta \ x^{\theta} - 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

based on a sample of size n.

20. Explain the procedure for testing the equality of two population proportions.

(6×2=12)

1.1

M 3545

PART-C

-3-

Answer any two questions.

(Weightage 4 each)

21. Let x₁, x₂ ... x_n be a sample of size n from a population with density function

 $f(x) = \frac{1}{\sigma\sqrt{2\pi} x} e^{-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2}; x > 0$

Obtain the method of moments estimators of μ and σ^2 .

22. Let (x_1, x_2) be a sample of size 2 selected from a population

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} ; & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

To test H_n : $\theta = 2$ against H_1 : $\theta = 4$ it is decided to accept H_0 if $x_1 + x_2 \ge 9.5$ and to reject otherwise. Obtain the level of significance and power of the test.

23. The gain in weights of two random samples of 8 rats each fed on two different diets A and B are given below :

Diet A : 49	53	51	52	47	50	52	53
Diet B : 52	55	52	53	50	54	54	53

Examine whether the difference in the average gain in weights is significant (choose 5% level of significance).

24. Fit a Poisson distribution to the following data and list the goodness of fit

No. of mistakes in a page :	0	1	2	3	4	5	6
No. of pages :	275	72	30	7	5	2	1

(Choose 5% level of significance.)

 $(2 \times 4 = 8)$