M 11407

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS-Regular) Examination, November 2011
CORE COURSE IN MATHEMATICS
5 B05 MAT : Vector Analysis

Time: 3 Hours   Max. Weightage: 30

1. Fill in the blanks :

a) The curvature of a straight line is ___________

b) If $f(x, y) = x^y$, then $\frac{\partial f}{\partial y} =$ ___________

c) The value of the triple integral $\int_0^1 \int_0^2 \int_0^3 dx\, dy\, dz$ is ___________

d) If $\vec{F} = \nabla \phi$ then the value of $\int_A^B \vec{F} \cdot d\vec{r}$ is ___________ **(Weightage 1)**

Answer **any six** from the following. (**Weightage 1 each**)

2. Show that $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{j} + 4\hat{k}$ are orthogonal.

3. Find the length of one turn of the helix $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\,\hat{k}$.

4. Find all second order partial derivatives of $s(x, y) = \tan^{-1}(y/x)$.

5. Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$.

6. If $f(x, y, z) = x^2 + y^2 - 2z^2$, find ∇f at the point (1, 1, 1).

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7. Find the area of the region bounded by coordinates axes and the line $x + y = 2$.

8. Find the average value of $f(x, y) = x \cos(xy)$ over the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$.

9. Find Curl of $\vec{F} = \left(x^2 - y\right)\hat{i} + 4z\hat{j} + x^2\hat{k}$.

10. State Divergence theorem. **(Weightage 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**)

11. Express $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ as the sum of a vector parallel to $\vec{a} = 3\hat{i} - \hat{j}$ and a vector orthogonal to $\vec{a}$.

12. Parametrize the line segment joining the points (–3, 2, –3) and (1, –1, 4).

13. Show that the curvature of a circle of radius a is $\frac{1}{a}$.

14. Find the directions in which $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ increases most rapidly and decreases most rapidly at the point (1, 1).

15. Find the linearization of $f(x, y) = e^x \cos y$ at the point $\left(0, \frac{\pi}{2}\right)$.

16. Find an equation for the tangent to the epllipse $\frac{x^2}{4} + y^2 = 2$ at the point (–2, 1).

17. Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

18. Find the Jacobian of the transformation from spherical coordinates to rectangular Cartesian coordinates.

19. Find the flux of $\vec{F} = (x - y)\,\hat{i} + x\hat{j}$ across the circle $x^2 + y^2 = 1$ in the xy – plane.

20. Evaluate the integral $\int_C (xy\,dy - y^2\,dx)$, where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$. **(Weightage 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**)

21. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

22. Find the centroid of the region in the first quadrant that is bounded above by the line $y = x$ and below by the parabola $y = x^2$.

23. Find the volume of the upper region D cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \pi/3$.

24. Find the centre of mass of a thin shell of constant density δ cut from the cone $z = \sqrt{x^2 + y^2}$ by the planes $z = 1$ and $z = 2$.

25. Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} + xy\,\hat{j}$ integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0$, $y = 0$, $x = a$, $y = a$.

(Weightage 3×3=9)
