Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.)
Examination, May 2016
Core Course in Mathematics
6B13 MAT : INTEGRAL TRANSFORMS

Time : 3 Hours   Max. Weightage : 30

1. Fill in the blanks :

   a) Laplace transform of t is _____

   b) A function without fundamental period is _____

   c) Example for an odd function is _____

   d) $Z(k) = $ _____   (Weightage 1)

Answer **any six** from the following (Weightage **1 each**).

2. State linearity property of Laplace transform.

3. Find $L(te^{-3t})$.

4. Find the inverse Laplace transform of $\dfrac{4}{s^2-2s-3}$.

5. Explain the convergence of Fourier series expansion of functions.

6. State second shifting theorem for Z-transform.

7. Find Z-transform of $n(n-1)$.

8. Find Z-transform of $e^{2(t+T)}$.

9. State Fourier integral theorem.

10. Find Fourier cosine transform of

$$f(x) = \begin{cases} x & 0 < x < a \\ 0 & x > a \end{cases}$$

(Weightage 6×1=6)

Answer **any seven** from the following (Weightage **2 each**).

11. Define Dirac's delta function. Also find its Laplace transform.

12. Find the inverse Laplace transform of $\log\left(\dfrac{s+1}{s-1}\right)$.

13. Find the Fourier series expansion of e^{-x} in the interval $(-l, l)$.

14. Express $f(x) = x^2$ as a half range sine series in $0 < x < \pi$.

15. Obtain the complex form of the Fourier series formula.

16. State and prove convolution theorem for Z-transforms.

17. Find the Z-transform of $f * g$ where $f(n) = 3^n$ and $g(n) = \cos n\theta$.

18. Using power series method, find the inverse Z-transform of $\log\left(\dfrac{z}{z+1}\right)$.

19. Prove that $\displaystyle\int_0^\infty \frac{w\sin xw}{k^2+w^2}\, dw = \frac{\pi}{2}e^{-kx}$, $x > 0$, $k > 0$.

20. Let $f(x)$ be continuous on the x-axis, $f(x) \to 0$ as $|x| \to \infty$ and $f'(x)$ be absolutely integrable on the x-axis, then prove that
$$F\{f''(x)\} = -w^2 F\{f(x)\}.$$ **(Weightage 7×2=14)**

Answer **any three** from the following (Weightage **3 each**).

21. If $f(t)$ is a periodic function with period T, then prove that
$$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

22. Using Laplace transform, solve the initial value problem :
$$y''' - 3y'' + 3y' - y = t^2 e^t, \quad y(0) = 1, \, y'(0) = 0, \, y''(0) = -2.$$

23. Obtain the Fourier series for the function $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$

Deduce that $\dfrac{1}{1^2} + \dfrac{1}{3^2} + \dfrac{1}{5^2} + \ldots = \dfrac{\pi^2}{8}$.

24. a) Find Z-transform of $r^n \sin n\theta$.

b) Find the inverse Z-transform of $\dfrac{z^3 - 20z}{(z-2)^3 (z-4)}$.

25. Find the Fourier transform of $f(x) = \begin{cases} 1-x^2, & |x| < 1 \\ 0, & |x| < 1 \end{cases}$.

Hence evaluate $\displaystyle\int_0^\infty \dfrac{x\cos x - \sin x}{x^3} \cos\left(\dfrac{x}{2}\right) dx$.

(Weightage 3×3=9)