K17U 0114

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2017
CORE COURSE IN MATHEMATICS
6B11 MAT : Complex Analysis
(2009 – 2013 Admns.)

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks (weightage **one**) :

a) The real part of $f(z) = z^3 + z + 1$ is________

b) If $f : \mathbb{C} \to \mathbb{C}$, then $\lim_{z \to \infty} f(z) = w_0$ if and only if $\lim_{z \to 0} f(y_z) =$ ______

c) The singularities of $\frac{1}{\sin\left(\frac{\pi}{2}\right)}$ are

d) If the principal part in the Laurent's expansion of a function f (z) about z = a contains only m terms (m > 0 and finite), then z = a is called a ______ of f (z).

(Wt. : 1)

Answer **any 6** questions from the following **9** questions : **(Weightage 1 each)**

2. Express in the polar form $\left(\frac{6+8i}{4-3i}\right)^2$.

3. Show that $\left|(2\bar{z}+5)(\sqrt{2}-i)\right| = \sqrt{3}\,|2z+5|$, where z = x + iz.

4. Show that $f(z) = \bar{z}$ is not differentiable.

5. Show that u (x, y) = 2x (1 – y) is harmonic.

6. If z_1 and z_2 are any two complex-numbers, show that $2\sin z_1 \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2)$.

P.T.O.

7. Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z| = 3$.

8. Prove that $\cos^{-1}(z) = -i\log\left[z + i(1-z^2)^{1/2}\right]$.

9. State the Cauchy's Residue theorem.

10. Find the residues of $f(z) = \frac{z^2 - 1}{(z+1)^2(z^2+4)}$ at the poles. **(Wt. : 1×6=6)**

Answer **any 7** questions from the following **10** questions. **(Weightage 2 each)**

11. Prove that an analytic function of constant absolute value is a constant.

12. If f (z) = u (x, y) + iv (x, y) and $\overline{f(z)} = u(x, y) - iv(x, y)$ is analytic in D, show that f (z) is a constant in D.

13. If w (t) , a complex valued function of a real variable, is integrable on [a, b], show that $\left|\int_a^b w(t)\, dt\right| \leq \int_a^b |w(t)|\, dt$.

14. Find $\int_C \bar{z} dz$, where C is the right hand half of the circle $|z| = 2$.

15. Find all the values of $(-8i)^{1/3}$.

16. Expand $\frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for $1<|z|<3$.

17. If a function f (z) is analytic inside and on a positively oriented circle C with centre at z_0 and radius R, show that $|f^n(z_0)| \leq \frac{n!M}{R^n} (n = 1, 2, 3, \ldots)$, where M is a positive real number such that $|f(z)| \leq M$.

18. If f (z) = u (x, y) + iv (x, y) is analytic in a domain D, show that the component functions u and v are harmonic in D.

19. If $z = z_0$ is a pole of order m of an analytic function f (z), show that $f(z) = (z - z_0)^m f(z)$, where $\delta(z)$ is analytic and non-zero at z_0.

20. If two functions p and q are analytic at a point z_0, $p(z_0) \neq 0$ and q has a zero of order m at z_0, show that the quotient $\frac{p(z)}{q(z)}$ has a pole of order m at z_0. **(Wt. : 2×7=14)**

Answer **any 3** questions from the following **5** questions (Weightage **3 each**).

21. If f (z) = u (x, y) + iv (x, y) and f′ (z) exists at a point $z_0 = x_0 + iy_0$, then show that u and v satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$ at (x_0, y_0).

22. If f (z) = u (x, y) + iv (x, y), $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$, show that $\lim_{z \to z_0} f(z) = w_0$ if and only if $\lim_{(x,y) \to (x_0, y_0)} u(x, y) = u_0$ and $\lim_{(x,y) \to (x_0, y_0)} v(x, y) = v_0$.

23. State and prove the fundamental theorem of algebra.

24. If a function f (z) is analytic everywhere inside and on a simple closed curve C taken in positive sense, prove that $f'(z) = \frac{1}{2\pi i} \int_C \frac{f(s)}{(s - z)^2} ds$, where z is interior to C.

25. If a function f (z) is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed curve C, then prove that

$$\int_C f(z)dz = 2\pi i \operatorname*{Res}_{z=0} \left[\frac{1}{z^2} f\left(\frac{1}{z} \right) \right].$$

(Wt. : 3×3=9)