K23P 1252

Reg. No. : ...............................

Name : ...................................

# I Semester M.Sc. Degree (CBSS – Regular) Examination, October 2022
## (2022 Admission)
## STATISTICS WITH DATA ANALYTICS
## MST1C02 : Probability Theory

Time : 3 Hours Max. Marks : 80



PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. Define a random variable. Explain the difference between a discrete and a continuous random variable.

2. Define the limit superior and limit inferior of a set of real numbers. Prove that if $A \subset B$, then $\lim\sup A \subseteq \lim\sup B$ and $\lim\inf A \subseteq \lim\inf B$.

3. Define the Cumulative Distribution Function (CDF) of a random variable. State the properties of a CDF.

4. State and prove the Jensen's inequality for convex functions.

5. Define convergence in probability of a sequence of random variables.

6. State the theorem that establishes the relationship between complete convergence and convergence in probability.

7. State the weak law of large numbers and its significance in probability theory.

8. State Lyapunov Central limit theorem. (8×2=16)

P.T.O.

PART – B

Answer **any 4** questions. **Each** question carries **4** marks.

9. Define field and sigma fields of sets. Prove that the intersection of two fields is a field.

10. State and prove Bayes' theorem for a finite number of events.

11. State the Jordan decomposition theorem for a real-valued function. Decompose the following cumulative distribution function using Jordan decomposition theorem.

$$F_{X(x)} = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & x = 0 \\ \frac{1}{2} + \frac{x}{2} & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

12. State and prove the Markov's inequality for non-negative random variables.

13. State and prove the Borel-Cantelli lemma for almost sure convergence.

14. State and prove Kolmogorov's WLLN numbers. **(4×4=16)**

PART – C

Answer **any 4** questions. **Each** question carries **12** marks.

15. A) State and prove the Borel 0-1 law for events in a Borel sigma-algebra.

    B) Define characteristic. Show the $n^{th}$ moment of a random variable using its characteristic function.

16. Let $X_1, X_2, ...., X_n$ be a set of exchangeable RVs. Then

$$E\left\{\frac{X_1 + X_2 + .... + X_k}{X_1 + X_2 + ... + X_n}\right\} = \frac{k}{n}, 1 \le k \le n.$$

17. For any characteristic function $\phi(u)$, prove that

a) $\text{Real}(1-\phi(u)) \geq \frac{1}{4}\text{Real}(1-\phi(2u))$

b) $|\phi(u)-\phi(u+h)|^2 \leq 2\{1-\text{Real}(\phi(h))\}$

c) $\int_{|x|<\frac{1}{u}} x^2 dF(x) \leq \frac{3}{u^2}\{1-\text{Real}(\phi(u))\}$

18. A) Prove that if $(X_n)$ converges to $(X)$ in probability and $(Y_n)$ converges to $(c)$ in probability, then $(X_n + Y_n)$ converges to $(X + c)$ in probability.

B) Prove that if $(X_n)$ converges to $(X)$ in $(r)^{th}$ mean and $(Y_n)$ converges to $(Y)$ in $(r)^{th}$ mean, then $(X_n + Y_n)$ converges to $(X + Y)$ $(r)^{th}$ mean.

19. State and prove Kintchine law of large numbers.

20. Define WLLN. Let $\{Y_n\}$ be a sequence of independent random variates with $P(X_k = \pm 2^k) = \frac{1}{2^{k+1}}$ and $P(X_k = 0) = 1 - \frac{1}{2^k}$; does $\{Y_n\}$ obey WLLN ? **(4×12=48)**