

**First Semester FYUGP Mathematics Examination
NOVEMBER 2024 (2024 Admission onwards)
KU1DSCMAT113 (FUNCTIONS, CALCULUS AND
MATRICES)**
(DATE OF EXAM: 4-12-2024)

Time : 120 min

Maximum Marks : 70

Part A (Answer any 6 questions. Each carries 3 marks)

1. Use the laws of exponents to simplify the following expressions:
(a) $2^{\sqrt{3}} \cdot 7^{\sqrt{3}}$
(b) $\left(\frac{2}{\sqrt{2}}\right)^2$ 3
2. Find the inverse of $y = \frac{x}{2} + 1$. 3
3. Evaluate $\lim_{x \rightarrow 0} \frac{1+x+\sin x}{3 \cos x}$. 3
4. Evaluate $\int a^{2x} dx$. 3
5. Evaluate $\int_0^{\frac{\pi}{4}} \sec^2 x dx$. 3
6. Find the transpose of
$$\begin{bmatrix} 1 & 5 & 6 & 3 \\ 2 & 5 & 7 & 8 \\ 5 & 9 & 2 & 4 \end{bmatrix}$$
 3
7. Define elementary matrices. 3
8. Find the rank of the matrix
$$A = \begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$$
 3

Part B (Answer any 4 questions. Each carries 6 marks)

9. Express the following logarithms in terms of $\ln 5$ and $\ln 7$
(a) $\ln(1/125)$ (b) $\ln(9.8)$ (c) $\ln(7\sqrt{7})$. 6
10. If $\sqrt{5-2x^2} \leq f(x) \leq \sqrt{5-x^2}$ for $-1 \leq x \leq 1$, find $\lim_{x \rightarrow 0} f(x)$, using the Sandwich Theorem. 6
11. If $2-x^2 \leq g(x) \leq 2 \cos x$ for all x , find $\lim_{x \rightarrow 0} g(x)$, using the Sandwich Theorem. 6

12. Express the matrix A as the sum of a symmetric and a skew-symmetric matrix where

$$A = \begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & -7 \end{bmatrix}.$$

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13. Express the matrix A as the sum of a symmetric and a skew-symmetric matrix where

$$A = \begin{bmatrix} a & a & b \\ c & b & b \\ c & a & c \end{bmatrix}.$$

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14. Find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

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Part C (Answer any 2 question(s). Each carries 14 marks)

15. a) Suppose u and v are functions of x that are differentiable at $x = 0$ and that $u(0) = 5, u'(0) = -3, v(0) = -1, v'(0) = 2$. Find the values of the following derivatives at $x = 0$.

(i) $\frac{d}{dx}(uv)$

(ii) $\frac{d}{dx}(7v - 2u)$.

b) Calculate the derivative $\frac{d}{dx}(\cos^{-1}(x^2))$.

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16. (a) Find the first and second derivative of the function $y = \frac{(x^2 + x)(x^2 - x + 1)}{x^4}$.

(b) Evaluate $\frac{d}{dx}(\ln \sin x)$.

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17. (a) Evaluate $\int \frac{(x+1)(x+\log x)^2}{2x} dx$.

(b) Evaluate $\int \sin^3 x dx$.

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