K25U 0161

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – OBE – Regular/Supplementary/ Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B13 MAT : Linear Algebra

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Define vector space.

2. What is the dimension of the vector space of all 2×2 symmetric matrices over $\mathbb{R}$?

3. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is linear, $T(1, 0) = (1, 4)$, and $T(0,1) = (2,5)$. What is $T(2, 3)$?

4. Find the eigenvalues of the matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

5. Let A be a 3×3 matrix, with eigenvalues 1, 0, 2. Find determinant of A.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. Let V be a vector space over $\mathbb{F}$. Show that for each element x in V, there exist a unique element y in V such that $x + y = 0$.

7. Let $M_{n \times n}(\mathbb{F})$ be the set of all $n \times n$-matrices over $\mathbb{F}$. Show that

   $S = \{A \in M_{n \times n}(\mathbb{F}) \mid tr(A) = 0\}$ is a subspace of $M_{n \times n}(\mathbb{F})$.

8. Let V be a vector space over $\mathbb{F}$. Show that $0.x = 0$ for each $x \in V$.

P.T.O.

9. Check whether the set $\{(1, -1, 2), (1, -2, 1) (1, 1, 4)\}$ is linearly independent or not.

10. Let $V = M(2, \mathbb{R})$, the set of all 2×2-matrices over $\mathbb{R}$ and let $W = \left\{ A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \in V \middle| a_{11} + a_{12} = 0 \right\}$. Find a basis of W.

11. Find the rank of a matrix A, where $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}$.

12. Show that $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (x, x + y)$ is a linear transformation.

13. Explain the condition for consistency and nature of solution of a non-homogeneous linear system of equations $Ax = B$.

14. Let A be a 2×3 matrix and B be a 3×3 matrix with rank (A) = 2 and rank (B) = 3. Find rank (AB).

15. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$.

16. Prove that the eigenvalues of an idempotent matrix are either zero or unity.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Using example, show that union of two subspaces of a vector space need not be a subspace.

18. If V is a vector space generated by a finite set S, then show that some subset of S is a basis for V.

19. Find a basis and dimension of the subspace $W = \{A \in M_{2\times 2}(\mathbb{R}) | tr(A) = 0\}$ of $M_{2\times 2}(\mathbb{R})$.

20. Let $T : V \to W$ be a linear transformation. Show that T is one-one if and only if $N(T) = \{0\}$.

21. Let $T : P_2(\mathbb{R}) \to P_3(\mathbb{R})$ be a linear transformation defined by $T(f(x)) = (2 - x)f(x)$. Find the matrix of T with respect to the standard basis of P_2 and P_3.

22. Let T be the linear operator on $\mathbb{R}^3$, the matrix of which in the standard ordered basis is $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ find T(x, y, z).

23. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and find its inverse.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(6×2=12)**

24. Let W_1 and W_2 be subspaces of a vector space V. Prove that V is the direct sum of W_1 and W_2 if and only if each vector in V can be uniquely expressed as $x_1 + x_2$, where $x_1 \in W_1$ and $x_2 \in W_2$.

25. Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Then show that $S \cup \{v\}$ is linearly dependent if and only if $v \in$ Space(S).

26. Let V and W be vector spaces and $T : V \to W$ be linear. Prove that N(T) and R(T) are subspaces of V and W respectively.

27. Find the characteristic equation of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and hence find its inverse.