K23N 0420

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, October 2023
(2022 Admission Onwards)
STATISTICS WITH DATA ANALYTICS
MST1C01 : Mathematical Methods for Statistics

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **2** marks :

1. Define vector space. Give one example.
2. Show that trace of a matrix is equal to the sum of its eigenvalues.
3. State Cayley-Hamilton theorem.
4. Define rank and signature of a quadratic form.
5. Define compactness of a metric space.
6. State the condition for a function f to be integrable with respect to α.
7. Show that $f_n(x) = \frac{x}{1+nx^2}$ is uniformly convergent on $\mathbb{R}$.
8. Find $\lim_{(x,y)\to(0,0)} (x+y)$. **(8×2=16)**

PART – B

Answer **any four** questions. **Each** question carries **4** marks :

9. Explain linear dependency and independency of vectors over a field. Examine whether the following vectors are linearly independent. {(1, 1, 1), (1, 3, 2), (2, 1, 1)}.
10. Show that r(A′ A) = r(A), where r(A) denotes the rank of A.

P.T.O.

11. Show that a real symmetric matrix A is positive definite if and only if there exists a non-singular matrix Q such that A = Q′ Q.

12. State and prove Rolle's theorem.

13. Find $\int_0^2 [x]\, d(x^2)$, where [x] is the greatest integer function of x.

14. Examine the convergence of the improper integral $\int_0^1 \frac{dx}{x^2}$. **(4×4=16)**

PART – C

Answer **any four** questions. **Each** question carries **12** marks :

15. i) If A and B are matrices of same type, then show that $r(A + B) \leq r(A) + r(B)$, where r(A) denotes the rank of A.

 ii) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{bmatrix}$.

16. i) If A is non-singular, prove that eigenvalues of A^{-1} are the reciprocal of eigenvalues of A.

 ii) Show that the characteristic roots of an idempotent matrix are either zero or unity.

17. i) Define a quadratic form. What are the types of quadratic forms ?

 ii) Obtain the matrix corresponding to the following quadratic form and classify it. $x^2 + 2y^2 + 3z^2 + 4xy + 5yz + 6zx$.

18. i) What are the types of discontinuities of a function ? Illustrate through suitable examples.

 ii) Define Reimann-Stieltjes integral and state and prove the linearity property.

19. i) Show that $\lim_{(x, y)\to(0, 0)} \frac{2xy}{x^2 + y^2}$ does not exist.

 ii) Show that every absolutely convergent integral is convergent.

20. State and prove Cauchy's criterion for uniform convergence. **(4×12=48)**