K23U 2366

Reg. No. : .....

Name : .....

V Semester B.Sc. Degree (C.B.C.S.S.-O.B.E.-Regular/Supplementary/ Improvement) Examination, November 2023
(2019 – 2021 Admissions)
CORE COURSE IN MATHEMATICS
5B06 MAT : Real Analysis – I

Time : 3 Hours Max. Marks : 48

PART – A

Answer any 4 questions. They carry 1 mark each. (4×1=4)

1. State Triangle Inequality.
2. Find $\lim\left(1+\frac{1}{2n}\right)^n$.
3. Define m-tail of a sequence.
4. Define continuity of a function at a point.
5. Define Rearrangement of the series.

PART – B

Answer any 8 questions from among questions 6 to 16. These questions carry 2 marks each. (8×2=16)

6. Determine the set A of $x \in R$ such that $|2x + 3| < 8$.
7. If $a \in R$ and $a \neq 0$ then show that $a^2 > 0$.
8. Discuss the convergence of $\lim\left(\frac{n}{2^n}\right)$.
9. Find the limit of the sequence whose terms are given by $x_1 = 8$, $x_{n+1} = \frac{x_n}{2} + 2$ for $n \in N$.
10. State Monotone Convergence Theorem.
11. Define subsequence of a sequence with an example.
12. State Alternating Series test.

P.T.O.

13. Define convergent Series.
14. If $\sum a_n$ with $a_n > 0$ is convergent, then is $\sum \sqrt{a_n}$ always convergent. Justify.
15. Show that $f(x) = \frac{1}{x}$ defined on $A = (0, \infty)$ is unbounded on A.
16. State Boundedness Theorem.

PART – C

Answer **any 4** questions from among questions 17 to 23. These questions carry **4 marks each**. (4×4=16)

17. Show that cosine function is continuous on R.
18. Discuss the convergence of $\sum_{n=0}^{\infty} r^n$, $r \in R$, $|r| < 1$.
19. Discuss the convergence of $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.
20. Discuss the convergence of the sequences
    a) $((-1)^n)$ and
    b) (n).
21. Show that Cauchy sequence of real numbers is bounded.
22. State and prove Archimedean property.
23. If a and b are positive real numbers, $a \neq b$ then show that $\sqrt{ab} \leq \frac{(a+b)}{2}$.

PART – D

Answer **any 2** questions from among questions **24** to **27**. These questions carry **6 marks each**. (2×6=12)

24. State and prove density theorem of rational numbers in R.
25. State and prove Squeeze theorem for sequences. Hence find $\lim\left(\frac{\sin n}{n}\right)$.
26. Discuss the convergence of
    a) $\sum_{n=0}^{\infty} \frac{1}{(n+1)(n+2)}$
    b) $\sum_{n=1}^{\infty} \frac{(\cos n)}{n^2}$.
27. Discuss the continuity of
    a) Dirichlet's function
    b) Thomae's function.