



K22U 1561

Reg. No. :

Name :



IV Semester B.Sc. Degree CBCSS (OBE) Regular/Supplementary/
Improvement Examination, April 2022
(2019 Admission Onwards)
CORE COURSE IN MATHEMATICS
4B04 MAT : Number Theory and Applications of Integrals

Time : 3 Hours

Max. Marks : 48

PART – A

(Short Answer Type)

Answer **any four** out of five questions. **Each** question carries **1** mark.

1. Define greatest common divisor of two integers with atleast one integer is non zero.
2. State the Fundamental Theorem of Algebra.
3. Find the quotient q and remainder r when 3054 is divided by 162.
4. State Fermat's theorem.
5. If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$, then show that $ac \equiv bd \pmod{n}$. (4×1=4)

PART – B

(Short Essay Type)

Answer **any eight** out of eleven questions. **Each** question carries **2** marks.

6. Prove or disprove : If $a \nmid (b + c)$, then either $a \nmid b$ or $a \nmid c$.
7. If p is a prime and $p \mid ab$, then show that $p \mid a$ or $p \mid b$.
8. Find the gcd (306, 657).
9. For $n \geq 1$, show that $7/5^{2n} + 3 \cdot 2^{5n-2}$.

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10. If $\gcd(a, 35) = 1$, show that $a^{12} \equiv 1 \pmod{35}$.
11. Evaluate $\int_0^3 \sqrt{y+1} dy$.
12. Evaluate $\int_{\pi/4}^{\pi/2} \cot \theta \csc^2 \theta d\theta$.
13. Find the length of the curve $y = \frac{4\sqrt{2}}{3} x^{3/2} - 1$, $0 \leq x \leq 1$.
14. A pyramid 3 m high has a square base that is 3 m on a side. The cross section of the pyramid perpendicular to the altitude x m down from the vertex is a square x m on a side. Find the volume of the pyramid.
15. The region between the curve $y = \frac{\sqrt{x}}{2}$, $0 \leq x \leq 8$ and the x -axis is revolved about the x -axis to generate a solid. Find its volume.
16. Find the volume of the solid generated by revolving the region between the y -axis and the curve $x = \frac{1}{y}$, $1 \leq y \leq 8$, about the y -axis. (8×2=16)

PART – C

(Essay Type)

Answer **any four** out of seven questions. **Each** question carries **4** marks.

17. Show that the number $\sqrt{2}$ is irrational.
18. Find the remainder obtained upon dividing the sum $1! + 2! + 3! + \dots + 99! + 100!$ by 12.
19. By using Euler's theorem, find the last two digits in the decimal representation of 3^{256} .
20. Find the area of the region enclosed by the parabola $y = x^2$ and the line $y = -x^2 + 4x$.
21. Find the length of the graph of $y = \frac{1}{2}(e^x + e^{-x})$, $0 \leq x \leq 2$.



22. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$.
23. The region bounded by the parabola $y = x^2$ and the line $y = 2x$ in the first quadrant is revolved about the y -axis to generate a solid. Find the volume of the solid.

(4×4=16)

PART – D

(Long Essay Type)

Answer **any two** out of four questions. **Each** question carries **6** marks.

24. If a cock is worth 5 coins, a hen 3 coins and three chicks together 1 coin, how many cocks, hens and chicks, totalling 100 can be bought for 100 coins ?
25. Prove that the integers $53^{103} + 103^{53}$ is divisible by 39 and that $111^{333} + 333^{111}$ is divisible by 7.
26. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.
27. Find the lateral surface area of the cone generated by revolving the line segment $y = \frac{x}{2}$, $0 \leq x \leq 4$ about the x -axis.

(2×6=12)