K24P 3959

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, October 2024
(2022 Admission Onwards)
STATISTICS WITH DATA ANALYTICS
MST1C02 : Probability Theory

Time : 3 Hours Max. Marks : 80

PART – A

(Answer **all** questions. **Each** question carries **2** marks.)

1. Define limit of a sequence of sets.
2. Show that arbitrary intersection of σ-fields is a σ-field.
3. State Jordan decomposition theorem.
4. Determine whether $1 - it$ is a characteristic function or not.
5. Prove that if $X_n \xrightarrow{p} X$ and $Y_n \xrightarrow{p} Y$, then $X_n + Y_n \xrightarrow{p} X + Y$.
6. State Helly-Bray lemma.
7. State Demoivre-Laplace central limit theorem.
8. Determine whether weak law of large numbers hold for the sequences of random variables $\{X_n\}$ with $P\left[X_k = \pm 2^k\right] = \frac{1}{2}$. **(8×2=16)**

PART – B

(Answer **any four** questions. **Each** question carries **4** marks.)

9. Define Borel field with an example. Is it a sigma field ?
10. State and prove continuity property of a probability measure.

P.T.O.

11. Define expectation of a random variable. Prove the linearity and scale preserving property of it.

12. State and prove Jensen's inequality.

13. Prove or disprove $X_n \xrightarrow{P} X$ implies $X_n \xrightarrow{a.s.} X$.

14. Establish Kolmogorov strong law of large numbers for a sequence of independent random variables. **(4×4=16)**

PART – C

(Answer **any four** questions. **Each** question carries **12** marks.)

15. a) Show that a σ-field is a monotone field and conversely.
    b) Prove or disprove : Every field is a σ-field.

16. Define a distribution function. Show that it is non-decreasing and right continuous.

17. State and prove basic inequality. Deduce Markov inequality from it.

18. a) State inversion theorem of a characteristic function.
    b) Show that characteristic function is real if and only if the distribution function is symmetric about zero.

19. a) Define convergence in r^{th} mean and almost sure convergence of a sequence of random variables.
    b) Prove that almost sure convergence implies convergence in probability.

20. State and prove a necessary and sufficient condition for weak law of large numbers to hold. **(4×12=48)**

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