

K21U 3468

Reg. No. :

Name :

II Semester B.Sc. Degree (CBCSS - OBE - Reg./Sup./Imp.) Examination, April 2021 (2019 Admission Onwards) CORE COURSE IN MATHEMATICS 2B02 MAT : Integral Calculus and Logic

Time : 3 Hours

Max, Marks: 48

PART - A

Short answer questions. Answer any 4.

- 1. Find $\int \frac{1}{\sqrt{4+x^2}} dx$ in terms of hyperbolic functions.
- 2. Convert the equation $r = \sec \theta$ in to cartesian form.
- 3. State Fubini's theorem (first form).
- 4. Write the contrapositive of the statement : $x > y \Rightarrow x^2 > y^2$.
- 5. If P and Q are two statements, when will be the statement $-(P \lor Q)$ true? (4×1=4)

PART - B

Short essay questions - Answer any 8.

- 6. Evaluate $\int_{0}^{\frac{\sqrt{3}}{2}} x \cos^{5}(2x^{2}) dx.$ 7. Evaluate $\int e^{x} \sinh 2x dx \cdot$
- 8. Express the Cartesian coordinates (x, y, z) in terms of the cylindrical coordinates (r, θ, z).
- 9. Evaluate $\iint_{R} \cos \theta \, dr \, d\theta$, where R is : $0 \le \theta \le \frac{\pi}{2}$; $0 \le r \le 3$.

P.T.O.

K21U 3468

-2-

sec 0

r°drd0.

10. Plot the domain of integration of the integral $\int_{0}^{1} \int_{1-x}^{1+x} f(x, y) dydx$.

11. Find $\int_{0}^{\infty} \sin x \, dx$ using trapezoidal rule, taking two sub-intervals.

12. Find $\int_{0}^{\pi} f(x) dx$ using Simpson's 1/3 rule, where the function f(x) is given by

x	0	1	2	3	4
f(x)	0	1	8	27	64

- 13. If x is an odd integer, prove that x² + 2 is odd.
- 14. Define the terms :
 - i) tautology
 - ii) negation of a statement.
- 15. State the two D'morgan's laws for quantified statements.
- 16. If p(x) : x is an integer multiple of 2 and q(x) : x is an integer multiple of 3, what is the statement corresponding to ¬ (p(x) ∧ q(x)) ?
 (8×2=16)

PART - C

Essay questions - Answer any 4.

- 17. Derive reduction formula for $\int \sec^n x \, dx$, n > 2.
- 18. Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{d}{dx}(\coth^{-1}x)$.
- 19. Express the integral in Cartesian form (no need to evaluate) ∫

- 20. Evaluate $\int_{A} \int_{A} dxdy$, where A is the region in the first quadrant bounded by the curve xy = 16 and the lines y = x, y = 0, x = 6.
- Using trapezoidal rule, find the approximate area bounded between the parabola y = 4 - x² and the X axis, using 8 sub-intervals.
- 22. Prove the following statement using the method of contradiction : "If n is an integer and n² is even, then n must be even". Is the converse true ? Justify.
- 23. Write the negation of the statements using quantifiers :

i)
$$(\forall x \in \mathbb{R}) (x^2 + 4x < 7)$$

ii) $(\exists x \in \mathbb{R}) (x^2 + 4x > 7)$.

PART – D

Long Essay Questions - Answer any 2.

24. Find the following integrals :

i)
$$\int_{1}^{2} \frac{1}{x \sqrt{9 + (\ln x)^2}} dx$$

ii)
$$\int \frac{\cosh^{-1} x}{\sqrt{x^2 - 1}} \, \mathrm{d}x \, \cdot$$

- 25. Evaluate $\iint_{V} \int_{V} \int_{V} 2x \, dz \, dy \, dx$, where V is the cylindrical solid $x^2 + y^2 = 4$, cut by the XY plane below and by the plane x + 2y + z = 9 above.
- 26. Find $\int_{0}^{6} \frac{5}{x+2} dx$ using Simpson's 1/3 rule, taking six sub-intervals.
- 27. i) Explain the three methods of proof.
 - ii) If x + a > 0 for all a > 0, prove that $x \ge 0$.

(2×6=12)

K21U 3468

 $(4 \times 4 = 16)$

-3-