K23U 0513

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CBCSS-OBE-Regular/Supplementary/ Improvement) Examination, April 2023
(2019 and 2020 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours   Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. State second form of the fundamental theorem of integral calculus.
2. State Lebesgue's integrability criterion.
3. Evaluate $\int_0^\infty \frac{dx}{1+x^2}$.
4. Evaluate $\int_0^\infty x^4 e^{-x}\, dx$.
5. Find the limit of the sequence of function $f_n(x) = x^n$ on [0, 1].

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Prove that $f(x) = \sqrt{x}$ is uniformly continuous on $[1, \infty)$.
7. State nonuniform continuity criteria.
8. If $f : A \to \mathbb{R}$ is a Lipschitz function, then prove that f is uniformly continuous on A.
9. Prove that every constant function on [a, b] is in $\mathcal{R}[a, b]$.
10. If $f(x) = x^2$, for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition $\dot{\mathcal{P}} = \{0, 1, 2, 4\}$ with tags at the left end points of the sub intervals.
11. Prove that the function $d(x, y) = |x - y|$ is a metric on $\mathbb{R}$.
12. Define closed set in a metric space. Give an example.
13. Investigate the convergence of $\int_0^1 \frac{1}{1-x}\, dx$.
14. Prove that $\int_1^\infty \frac{(1-e^{-x})}{x}\, dx$ diverges.

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15. Evaluate $\int_1^\infty \sqrt{x}e^{-x^2}dx$.

16. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and if they are both bounded on A, then their product f g is uniformly continuous on A.

18. If $f \in \mathcal{R}[a, b]$, then prove that f is bounded.

19. Evaluate $\int_0^1 \frac{dx}{(x-1)^{\frac{2}{3}}}$.

20. Prove that $B(m, n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$.

21. Prove that $\Gamma m \, \Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \cdot \Gamma(2m)$.

22. Show that the sequence of functions $\left(\frac{x^n}{1+x^n}\right)$ does not converge uniformly on [0, 2].

23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Then prove that f is continuous on A.

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. State and prove continuous extension theorem.

25. Prove that a function $f \in \mathcal{R}[a, b]$ if and only if for every $\epsilon > 0$ there exists $\eta_\epsilon > 0$ such that if $\dot{\mathcal{P}}$ and $\dot{\mathcal{Q}}$ are any two tagged partitions of [a, b] with $\|\dot{\mathcal{P}}\| < \eta_\epsilon$ and $\|\dot{\mathcal{Q}}\| < \eta_\epsilon$, then $\left| S(f, \dot{p}) - S(f, \dot{\mathcal{Q}}) \right| < \epsilon$.

26. Prove that if $f : [a, b] \to \mathbb{R}$ is monotone on [a, b], then $f \in \mathcal{R}[a, b]$.

27. State and prove Cauchy criterion for uniform convergence of sequence of functions.