K17P 1598

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree (Regular) Examination, October 2017
(2017 Admission)
MATHEMATICS
MAT 1C05 : Differential Equations

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any four** questions from Part **A**. **Each** question carries **4** marks.*
*2) Answer **any four** questions from Part **B** without omitting **any** Unit. **Each** question carries **16** marks.*

PART – A

1. Find the general solution of the differential equation $y'' + y = 0$, in terms of power series in x.

2. Determine the nature of the point x = 0 for

   i) $y'' + (\sin x)y = 0$

   ii) $x^2y'' + (\sin x)y = 0$

3. Define $J_p(x)$. If p is an integer $m \geq 0$ show that $J_{-m}(x) = (-1)^m J_m(x)$.

4. Replace the following differential equations by an equivalent system of first order equations

   i) $y^{(n)} = f(x, y, y', y'', \ldots, y^{(n-1)})$

   ii) $y''' = y'' - x^2(y')^2$.

5. Find the normal form of the Bessel's equation $x^2y'' + xy' + (x^2 - p^2)y = 0$.

6. Show that $f(x, y) = y^{1/2}$ does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$.

P.T.O.

PART – B

Unit – I

7. a) Find the general solution of $(1 + x^2)\ y'' + 2xy' - 2y = 0$ in terms of power series in x. Can you express this solution by means of elementary functions ?

   b) Find a solution of $y' = x - y,\ y(0) = 0$ as a power series in x. Express the series solution in terms of familiar functions. Verify your solution by solving the initial value problem directly.

8. a) Show that the equation $4x^2y'' - 8x^2y' + 4(x^2 + 1)y = 0$ has only one Frobenius solution. Also find the general solution.

   b) Find the general solution of the differential equation $2x^2\ y'' + x(2x + 1)y' - y = 0$, by using the method of Frobenius.

9. a) Define Gauss' hypergeometric equation and obtain the hypergeometric series as a solution of this equation.

   b) Define F(a, b, c, x) and show that $F'(a, b, c, x) = \frac{ab}{c} F(a + 1, b + 1, c + 1, x)$.

Unit – II

10. a) Obtain the solutions of the Legendre equation $(1 - x^2)\ y'' - 2xy' + n(n + 1)y = 0$, where n is a non-negative integer, bounded near x=1 in terms of hypergeometric functions.

    b) Prove the orthogonality property of Legendre polynomials :

$$\int_{-1}^{1} P_m(x)\, P_n(x) = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n \end{cases}.$$

11. a) Derive Bessel function of the first kind of order p. Where p is a non-negative constant as solution to the Bessel equation.

    b) Show that

       i) $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$ and

       ii) $\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$.

12. a) If $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ are two linearly independent solutions on [a, b] of the system $\frac{dx}{dt} = a_1 x + b_1 y$, $\frac{dy}{dt} = a_2 x + b_2 y$, then prove that the general solution of the system is $x = c_1 x_1(t) + c_2 x_2(t)$, $y = c_1 y_1(t) + c_2 y_2(t)$ for any constants c_1 and c_2.

b) Find the general solution of the system $\frac{dx}{dt} = x + y$, $\frac{dy}{dt} = 4x - 2y$.

Unit – III

13. a) If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of $y'' + P(x)y' + Q(x)y = 0$, then prove that the zeros of these functions are distinct and occur alternately in the sense that $y_1(x)$ vanishes exactly once between any two successive zeros of $y_2(x)$ and conversely.

b) State and prove the Sturm comparison theorem.

14. a) Find the exact solution of the initial value problem $y' = x + y$, $y(0) = 1$. Starting with $y_0(x) = 1$, calculate $y_1(x)$, $y_2(x)$ and $y_3(x)$. Also show that $y_n(x)$ converges to the exact solution.

b) Give an example of an initial value problem having more than one solution and give explanation for the non-uniqueness of solutions.

15. a) Let f(x, y) be a continuous function that satisfies a Lipschitz condition on a strip defined by $a \le x \le b$ and $-\infty < y < \infty$. If (x_0, y_0) is any point of the strip, then prove that the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ has a unique solution on the interval $a \le x \le b$.

b) For what points (x_0, y_0) does the initial value problem $y' = y\,|\,y\,|$, $y(x_0) = y_0$ has a unique solution on some interval $|\,x - x_0\,| \le h$?