K23P 0502

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (C.B.S.S. – Reg./Supple./Imp.) Examination, April 2023
(2019 Admission Onwards)
MATHEMATICS
MAT 2C 10 : Partial Differential Equations and Integral Equations

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any 4** questions. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from the equation $F(z - xy, x^2 + y^2)$ and find the corresponding Partial differential equation.
2. Show that $z = ax + \frac{y}{a} + b$ is complete integral of $pq = 1$.
3. State and prove maximum principle for harmonic function.
4. Prove that the solution of Neumann problem is unique up to the addition of a constant.
5. Define Fredholm integral equation of second kind and give an example.
6. Solve the integral equation $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi$ by iterative method.

PART – B

Answer **any 4** questions without omitting **any** Unit. **Each** question carries **16** marks.

Unit – I

7. a) Find the general integral of the equation
   $y^2 p - xyq = x(z - 2y)$.

   b) Prove that the equations $f = xp - yq - x = 0$, $g = x^2 p + q - xz = 0$ are compatible and find a one parameter family of common solutions.

8. a) Find the complete integral of $(p^2 + q^2)y - qz = 0$ by Charpit's method.

   b) Solve the PDE by Jacobi's method
   $z^2 + zu_x - u_x^2 - u_y^2 = 0$.

P.T.O.

9. a) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x_0(s) = 1$, $y_0(s) = 0$ and $z_0(s) = s$.

b) Find the characteristic strips of the equation $xp + yq = pq$ where the initial curve is $c : z = \frac{x}{2}, y = 0$.

Unit – II

10. a) Reduce the equation $u_{xx} - 4x^2u_{yy} = \frac{1}{x}u_x$ into a canonical form.

b) Derive d' Alemberts solution of wave equation.

11. a) Solve $y_{tt} - c^2y_{xx} = 0,\ 0 < x < 1,\ t > 0$

$y(0, t) = y(1, t) = 0$

$y(x, 0) = x(1 - x),\ 0 \le x \le 1$

$y_t(0, t) = 0,\ 0 \le x \le 1$

b) Prove that solution of Dirichlet problem is stable.

12. a) Solve the Dirichlet problem for a circle.

b) Solve the heat conduction problem in a finite rod.

Unit – III

13. a) Transform the boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0$, $y(0) = 0$, $y(l) = 0$ to an integral equation.

b) Show that the characteristic function of the symmetric Kernel corresponding to distinct characteristic numbers are orthogonal.

14. a) Using Green's function, solve the boundary value problem $y'' + xy = 1$, $y(0) = 0$, $y(l) = 1$.

b) Show that any solution of the integral equation $y(x) = \lambda\int_0^1 (1 - 3x\xi)y(\xi)\,d\xi + F(x)$ can be expressed as the sum of F(x) and some linear combination of the characteristic functions.

15. a) Show that the integral equation $y(x) = 1 + \frac{1}{\pi}\int_0^{2\pi} \sin(x + \xi)\,y(\xi)\,d\xi$ possess infinitely many solution.

b) Find the Resolvent Kernel for the Kernel $k(x, \xi) = xe^{\xi}$ in the interval $[-1, 1]$.