K20U 1831

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree CBCSS (OBE) – Regular Examination, November 2020
(2019 Admission Only)
CORE COURSE IN MATHEMATICS
3B03 MAT : Analytic Geometry and Applications of Derivatives

Time : 3 Hours

Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark.

1. Find the focus of the parabola $y^2 = 12x$.
2. Find all points of intersection of $x^2 = 4y$ and $y^2 = 4x$.
3. Let $f(x) = |x^3 - 9x|$. Does $f'(3)$ exists ?
4. Define radius of curvature of a curve at any point.
5. Find the length of the perpendicular from (0, 0) on the line $x \tan t + y - a \sin t = 0$ where t is a parameter. **(4×1=4)**

PART – B

Answer **any eight** questions. **Each** question carries **two** marks.

6. Identify the conic $r = \frac{4}{2 - 2\cos\theta}$.
7. Find the polar equation for the circle $(x - 6)^2 + y^2 = 36$.
8. Verify the Lagrange's theorem for the function $f(x) = e^x$ in [0, 1].
9. Find the asymptotes of $(x^2 - a^2)(y^2 - b^2) = a^2b^2$.
10. Find the angle of intersection of the curves $x^2 = 4y$ and $y^2 = 4x$ at (4, 4).

P.T.O.

11. Evaluate $\lim_{x \to 2} \frac{x-2}{x^2-4}$.

12. Find the absolute maximum value of the function $f(x) = x^2 - 1, -1 \leq x \leq 2$.

13. Expand sin x in powers of $x - \pi$.

14. Find the radius of curvature ρ at the origin for the curves $y^4 + x^3 + a(x^2 + y^2) - a^2y = 0$.

15. Prove that if f has a local maximum value at an interior point c of its domain and if f′(c) is defined at c, then f′(c) = 0.

16. For the cardioid, $r = a(1 - \cos\theta)$, prove that length of polar subtangent is $2a \sin^2 \frac{\theta}{2} \tan \frac{\theta}{2}$. (8×2=16)

PART – C

Answer **any four** questions. **Each** question carries **four** marks.

17. What is an ellipse ? Find its standard form equations centered at the origin.

18. Find the polar equation in the form $r \cos(\theta - \theta_0) = r_0$ of the line $\sqrt{2}x + \sqrt{2}y = 6$.

19. Find the equation of the tangent line of the curve $y(x - 2)(x - 3) - x + 7 = 0$ at the point where it cuts the x-axis.

20. Prove that the curves $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$ intersects at right angle.

21. Show that the sum of the intercepts on the axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = a$ is a constant.

22. Find the critical point of $f(x) = x^{\frac{1}{3}}(x - 4)$. Identify the interval on which f is increasing and decreasing.

23. If $f(x) = \log(1 + x), x > 0$ using Maclaurin's theorem, show that for $0 < \theta < 1$

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3(1+\theta x)^3}.$$

(4×4=16)

PART – D

Answer **any two** questions. **Each** question carries **6** marks.

24. Find the Cartesian equation for the hyperbola centered at the origin that has focus at (3, 0) and the line x = 1 as the corresponding directrix. Sketch the graph.

25. Find the lengths of the tangent, normal, subtangent and subnormal for the cycloid $x = a(1 + \sin t)$, $y = a(1 - \cos t)$.

26. Find the coordinates of the centre of curvature at any point of the parabola $y^2 = 4ax$. Hence show that its evolute is $27ay^2 = 4(x - 2a)^3$.

27. Sketch the graph of $f(x) = \dfrac{(x+1)^2}{1+x^2}$. **(2×6=12)**

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