K24P 3958

Reg. No. : ....

Name : ....

I Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, October 2024 (2022 Admission Onwards) STATISTICS WITH DATA ANALYTICS MST1C01 : Mathematical Methods for Statistics

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **2** marks.

1. Define linear dependence and independence of vectors.

2. Define orthogonal matrix. Give an example.

3. Evaluate the determinant of the matrix $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 2 \\ -2 & 0 & 1 \end{bmatrix}$.

4. Define non-negative definiteness of a matrix. Give one example.

5. Define connectedness of a metric space.

6. What are monotone functions ? Give an example.

7. Define beta and gamma functions.

8. Find $\lim_{(x,y)\to(1,2)} (x^2 + 2y)$. **(8×2=16)**

PART – B

Answer **any four** questions. **Each** question carries **4** marks.

9. Check whether the following system of equations are consistent :

$2x_2 - x_3 = 1$

$x_1 - x_2 + 3x_3 = 2$

$x_1 + x_2 + 2x_3 = 5.$

P.T.O.

10. Prove that rank of a non-singular matrix is equal to the rank of its reciprocal (inverse) matrix.

11. Compute the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

12. Find $\int_0^2 x^2 \, dx^2$.

13. Examine the convergence of the improper integral $\int_0^1 \frac{dx}{x^2}$.

14. Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent in any interval [0, b], b > 0. **(4×4=16)**

PART – C

Answer **any four** questions. **Each** question carries **12** marks.

15. i) If A and B are two n rowed square matrices of the same type, then show that rank (A) ≥ rank(A) + rank(B) – n.

    ii) Show that the only idempotent matrix with full rank is the identity matrix.

16. i) State and prove Cayley-Hamilton theorem.

    ii) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 3 & -3 & 2 \\ 1 & 1 & -1 \end{bmatrix}$.

17. i) Write the following quadratic form in matrix notation :
    $2x_1^2 + x_2^2 - x_3^2 + 5x_1x_2 - 2x_1x_3 + x_2x_3$.

    ii) Show that the determinant of a positive definite matrix is positive.

18. Define Reimann-Stieltjes integral and state and prove the condition for a function *f* to be integrable with respect to α on [a, b].

19. State Weierstrass's M test. Show that $\sum \frac{\sin(x^2 + n^2 x)}{n(n+1)}$ is uniformly convergent for all real x.

20. State and prove Taylor's theorem for the function of several variables. **(4×12=48)**