K22U 1310

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/Supplementary/Improvement) Examination, April 2022
(2019 Admission Onwards)
COMPLEMENTARY ELECTIVE COURSE IN STATISTICS
2C02STA: Probability Theory and Random Variables

Time : 3 Hours Max. Marks : 40

***Instruction** : **Use** of calculators and statistical tables are **permitted**.*

PART – A

(Short Answer)

Answer **all 6** questions : **(6×1=6)**

1. Define random experiment.

2. Write down the axiomatic definition of probability.

3. If two unbiased dice are thrown, then what is the probability that the total of the numbers on the dice is 8 ?

4. Write down the conditions for the mutual independence of three events.

5. Distinguish between discrete and continuous random variables.

6. Define probability mass function. Write down its properties.

PART – B

(Short Essay)

Answer **any 6** questions : **(6×2=12)**

7. If P(A) = 0.29 and P(B) = 0.43. Also, A and B are mutually exclusive. Find $P(A \cap \bar{B})$.

8. Find the probability of selecting an ace, 10 of diamonds or two spades from a well shuffled ordinary pack of 52 cards.

9. A committee of 4 people is to be appointed from 3 officers from category A, 4 officers from category B, 2 officers from category C and 1 from category D. Find the probability of forming the committee such that there must be one from each category.

10. If $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then find $P(A|B)$ and $P(B|A)$.

11. If A and B are independent events, then prove that A and $\bar{B}$ are independent events.

12. Define prior probability and posterior probability.

13. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} k, & -2 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of k and $P(|X| > 1)$.

14. Let X be the number of heads shown up when tossing of a fair coin three times independently. Write down the probability mass function and distribution function of X.

PART – C
(Essay)

Answer **any 4** questions : **(4×3=12)**

15. Prove that $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$.

16. If A, B, C are any three arbitrary events such that P(A) = P(B) = P(C) = 0.25, $P(A \cap B) = P(B \cap C) = 0$ and $P(C \cap A) = 0.125$. Find the probability that at least one of the events A, B and C occurs.

17. Explain pair wise independence and mutual independence of events. Give an example of set of events which are pair wise independent but not mutual independent.

18. A discrete random variable X has the probability mass function

x	0	1	2	3
P(x)	k	3k	5k	k

What is the value of k ? Find the probability mass function and distribution function of $Y = X^2 + 2X$.

19. If the probability density function of continuous random variable X is given by

$$f(x) = \begin{cases} ax, & 0 \leq x < 1 \\ a, & 1 \leq x < 2 \\ 3a - ax, & 2 \leq x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Then find (i) the value of a, (ii) the distribution function of X, (iii) $P(X > 1.5)$.

20. Let X be a continuous random variable with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

Find the probability density of $Y = X^2$.

PART – D

(Long Essay)

Answer **any 2** questions : **(2×5=10)**

21. State and prove addition theorem on two events. Establish its extension to the case of three events.

22. a) State and prove Baye's theorem.

 b) Three machines A, B and C produce respectively 60%, 30% and 10% of the total number of items of a factory. The percentages of defective output of these machines are 2%, 3% and 4% respectively. An item is selected at random and is found to be defective. Find the probability that the item was produced by machine C.

23. If the joint PDF of (X, Y) is given by $f_{XY}(x, y) = x + y$, $0 \leq x, y \leq 1$, find the PDF of XY.

24. If the joint PDF of (X, Y) is given by $f(x, y) = 24y(1 - x)$, $0 \leq y \leq x \leq 1$. Find the marginal and conditional distributions.