K25U 0158

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree (C.B.C.S.S. – O.B.E. – Regular/ Supplementary/Improvement) Examination, April 2025
(2019 to 2022 Admissions)
CORE COURSE IN MATHEMATICS
6B10 MAT : Real Analysis – II

Time : 3 Hours Max. Marks : 48

PART – A

Answer **any four** questions. **Each** question carries **one** mark. **(4×1=4)**

1. Give an example of a Lipschitz Function.
2. Define Riemann integral of a function $f : [a, b] \to \mathbb{R}$.
3. State Additivity Theorem.
4. Define Beta function.
5. Find $\lim_{n \to \infty} \frac{x}{x+n}$ for all $x \in [0, 1]$.

PART – B

Answer **any eight** questions. **Each** question carries **two** marks. **(8×2=16)**

6. Define step function on [a, b]. Give an example of a step function on [0, 2].
7. Check whether $f(x) = \sin x$ is Lipschitz on $\mathbb{R}$. Justify your answer.
8. Let $f(x) = x^3$ for $x \in [0, 4]$, calculate the Riemann sum with respect to the partition $P = (0, 1, 2, 4)$, take tags at the left end point of the subintervals.
9. Show that every constant function on [a, b] is in $\mathcal{R}[a, b]$.

P.T.O.

10. Let $f \in R[a, b]$, and if $[c, d] \subseteq [a, b]$, show that the restriction of f to [c, d] is in R[c,d].

11. Show that $\int_0^1 \frac{1}{1-x} dx$ diverges.

12. Show that $\Gamma n = (n-1)\,\Gamma(n-1)$.

13. Compute $\Gamma(-3/2)$.

14. Let $f_n : \mathbb{R} \to \mathbb{R}$ be defined by $f_n(x) = \frac{\sin nx}{n}$. Find the pointwise limit of the sequence of functions (f_n).

15. Define a metric d on a set S.

16. State Dini's theorem.

PART – C

Answer **any four** questions. **Each** question carries **four** marks. **(4×4=16)**

17. Define uniformly continuous function. Show that $\sin\left(\frac{1}{x}\right)$ is not uniformly continuous on (0, 1).

18. Show that every continuous function on [a, b] is Riemann integrable.

19. Let $f, g \in R[a, b]$, if $f(x) \leq g(x)$ for all $x \in [a, b]$ then show that $\int_a^b f \leq \int_a^b g$.

20. Evaluate $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.

21. Show that $\Gamma n \Gamma(1-n) = \frac{\pi}{\sin n\pi}$.

22. Evaluate $\int_0^{\pi/2} (\sin x)^{8/3} (\sec x)^{1/2} dx$.

23. Let (f_n) be a sequence of continuous functions on a set $A \subseteq \mathbb{R}$ and suppose that (f_n) converges uniformly on A to a function $f : A \to \mathbb{R}$. Show that f is continuous on A.

PART – D

Answer **any two** questions. **Each** question carries **six** marks. **(2×6=12)**

24. a) Show that a function f is uniformly continuous on the interval (a, b) if and only if it can be defined at the endpoints a and b such that the extended function is continuous on [a, b].

    b) Show that $f(x) = \frac{e^x - 1}{x}$ is uniformly continuous on (0,1).

25. State and prove fundamental theorem of calculus (Second form).

26. a) Show that $\int_1^\infty \frac{1}{x^p}\,dx = \begin{cases} \frac{1}{p-1}, & p > 1 \\ \infty, & p \le 1 \end{cases}$.

    b) Investigate the convergence of $\int_1^\infty \frac{1-e^{-x}}{x}\,dx$.

27. Let (f_n) be a sequence of functions in $\mathcal{R}[a, b]$ and suppose that (f_n) converges uniformly on [a, b] to f. Show that $f \in \mathcal{R}[a, b]$ and $\int_a^b f = \lim_{n\to\infty}\int_a^b f_n$.